UNIT – I
RELATIONS & FUNCTIONS

Multiple Choice Questions(1 Mark)

1 Relation $R = \{(x, y) : x \leq y ,\ x, y \in \mathbb{Z}\}$ is
(a)Reflexive and Symmetric relation (b)Reflexive and Transitive relation
(c)Symmetric and Transitive relation (d)Equivalence relation

2 Which of the following relations defined on set $A = \{1, 2, 3\}$ is reflexive but neither symmetric nor transitive :
(a)$R = \{(1,1), (2,2), (3,3)\}$ (b)$R = \{(1,1), (2,2), (3,3), (1,2), (2,3)\}$
(c) $R = \{(1,2), (1,3), (2,3), (3,1), (2,1)\}$ (d) $R = \{(1,2), (2,3), (1,3), (2,1)\}$

3 Function defined by $f : R \to R , f(x) = x^3$ is :
(a)only one-one (b)only onto
(c)one-one and onto (d)neither one-one nor onto

4 Function defined by $f : \mathbb{R} \to \mathbb{R} , f(x) = x^2$ is :
(a)only one-one (b)only onto
(c)one-one and onto (d)neither one-one nor onto

5 Relation $R = \{(x, x), (y, y), (x, y), (y, x)\}$ defined on the set $A = \{x, y\}$ is :
(a)Only Reflexive relation (b)Only Symmetric relation
(c)Only Transitive relation (d)Equivalence relation

6 Relation $R = \{(x, y) : x < y ,\ x, y \in \mathbb{Z}\}$ is
(a)Only Reflexive relation (b)Only Symmetric relation
(c)Only Transitive relation (d)Equivalence relation

7 Relation $R = \{(x, y) :\ x < y^2 \text{ where } x, y \in \mathbb{R}\}$ is
(a)Reflexive but not symmetric (b)Symmetric and transitive but not Reflexive
(c)Reflexive and Symmetric (d)Neither reflexive nor symmetric nor transitive

8 If $A = \{1, 4, 9, 16, 25, \ldots\ldots\}$ then function defined by $f : \mathbb{Z} \to A , f(x) = x^2$ is
(a)only one-one (b)only onto
(c)function is not defined (d)neither one-one nor onto

9 If $A = \{0, 1, 4, 9, 16, 25, \ldots\ldots\}$ then function defined by $f : \mathbb{N} \to A , f(x) = x^2$ is
(a)one-one but not onto (b)onto but not one-one
(c)one-one and onto (d)neither one-one nor onto

10 Function $f: \mathbb{R} \to \mathbb{R} , f(x) = \frac{3-7x}{2}$ is:
(a)one-one but not onto (b)onto but not one-one
(c)one-one and onto (d)neither one-one nor onto

11 If $A = \{0, 1, 4, 9, 16, 25, \ldots\ldots\}$ then function defined by $f : \mathbb{Z} \to A , f(x) = x^2$ is
(a)one-one but not onto (b)onto but not one-one
(c)one-one and onto (d)neither one-one nor onto

12 If $A = \{1, 4, 9, 16, 25, \ldots\ldots\}$ then function defined by $f : \mathbb{N} \to A , f(x) = x^2$ is
(a)only one-one (b)only onto
(c)one-one and onto (d)neither one-one nor onto

13 Function $f: \mathbb{R} \to \mathbb{R} , f(x) = \frac{1}{x}$ is:
(a)one-one but not onto (b)onto but not one-one
(c)function is not defined (d)neither one-one nor onto

14 Relation $R = \{(x, y) :\ x \leq y^3 \text{ where } x, y \in \mathbb{R}\}$ is
(a)Reflexive but not symmetric (b)Symmetric and transitive but not Reflexive
(c)Reflexive and Symmetric (d)Neither reflexive nor symmetric nor transitive

15 Function defined by $f : \mathbb{Z} \to \mathbb{W} , f(x) = x^2$ is
(a)one-one but not onto (b)onto but not one-one
(c)one-one and onto (d)neither one-one nor onto

Fill in the blanks(1 Mark)

1) Identity relation is also ____________________
2) If R is a relation defined on the set A then R is a subset of ______________
3) If function f is defined as $f: A \to f(A)$ then f is a _______________ function.
4) If R is a relation from A to B then R is __________ of $A \times B$.
5) The relation R (defined on the set A) is called ______________ if $(x, x) \in R\ \forall\, x \in A$
6) The relation R (defined on the set A) is called ______________ if $(x, y) \in R \Rightarrow (y, x) \in R$ $\forall\, x, y \in A$
7) The relation R (defined on the set A) is called ______________ if $(x, y), (y, z) \in R \Rightarrow (x, z) \in R$ $\forall\, x, y, z \in A$
8) If for the numbers x_1, x_2 in the domain of the function f we have $f(x_1) = f(x_2)$ implies $x_1 = x_2$ then function is _________________

4 Marks Questions

1. Prove that the following relations defined on the set of integers $\mathbb{Z}$:
   (i) $R = \{(x, y) : x - y \text{ is an integer } \}$
   (ii) $R = \{(x, y) : 2x - 2y \text{ is an integer } \}$
   (iii) $R = \{(x, y) : x - y \text{ is divisible by } 4 \}$
   (iv) $R = \{(x, y) : x - y \text{ is divisible by } 3 \}$
   (v) $R = \{(x, y) : |x - y| \text{ is divisible by } 5 \}$
   (vi) $R = \{(x, y) : |x - y| \text{ is divisible by } 6 \}$

   are (a)reflexive (b)symmetric (c)transitive

2. For the following functions $f : R \to R$:

   (i) $f(x) = \frac{3x+5}{2}$
   (ii) $f(x) = \frac{2x-7}{4}$
   (iii) $f(x) = \frac{3-2x}{4}$
   (iv) $f(x) = \frac{4-3x}{5}$
   (v) $f(x) = \frac{6-5x}{7}$
   (vi) $f(x) = \frac{5x+7}{6}$

   show that these functions are one-one and onto.

UNIT – I

Inverse Trigonometric Functions

Multiple Choice Questions(1 Marks Questions)

1 $\cos^{-1}\left(\cos\frac{2\pi}{3}\right)$ is equal to :

(a) $\frac{\pi}{5}$ (b) $\frac{2\pi}{3}$ (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{3}$

2 $\sin^{-1}\left(\frac{1}{2}\right)$ is equal to :

(a)0 (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{3}$

3 $\cos^{-1}(0)$ is equal to :

(a)0 (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{3}$

4 $\tan^{-1}(1)$ is equal to :

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{3}$

5 If $y = \sin^{-1}(x)$ then x belongs to the interval :

(a)$(0, \pi)$ (b)$(-1, 1)$ (c) $[-1, 1]$ (d) $[0, \pi]$

6 $\sin^{-1}\left(\sin\frac{\pi}{3}\right)$ is equal to :

(a) $\frac{\pi}{5}$ (b) $\frac{2\pi}{3}$ (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{3}$

7 If $\cos^{-1} x = y$ then x belongs to

(a)$(0, 1)$ (b)$(-1, 1)$ (c)$[-1, 1]$ (d)$[0, 1]$

8 Principal value of $\cos^{-1}(1)$ is

(a)0 (b)$\frac{\pi}{2}$ (c)$\frac{\pi}{3}$ (d)$\frac{\pi}{6}$

9 Range of function $\sec^{-1}$ is :

(a)$[0, \pi] - \left\{\frac{\pi}{2}\right\}$ (b)$(0, \pi)$ (c)$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right) - \{0\}$ (d)$[0, \pi]$

10 Domain of function $\operatorname{cosec}^{-1}$ is :

(a)$[-1, 1]$ (b)$\mathbb{R} - (-1, 1)$ (c)$\mathbb{R}$ (d)$(-1, 1)$

11 Domain of the function $\tan^{-1}$ is :

(a)$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ (b)$\mathbb{R} - (-1, 1)$ (c)$\mathbb{R}$ (d)$(-1, 1)$

12 If $\tan^{-1} x = y$, then y belongs to the interval :

(a)$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ (b)$\mathbb{R} - (-1, 1)$ (c)$\mathbb{R}$ (d)$(-1, 1)$

4 Marks Questions

1. Find the values of :

(i) $7\cos^{-1}\left(\frac{1}{2}\right) + 12\tan^{-1}(1) - 4\sin^{-1}(-1)$

(ii) $5\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) - 3\tan^{-1}(\sqrt{3}) + 7\sin^{-1}\left(\frac{1}{2}\right)$

(iii) $5\cos^{-1}\left(-\frac{1}{2}\right) + 8\tan^{-1}(1) - 3\sin^{-1}\left(-\frac{1}{2}\right)$

(iv) $2\operatorname{cosec}^{-1}(-1) - 5\sec^{-1}\left(\frac{2}{\sqrt{3}}\right) + \sin^{-1}\left(-\frac{1}{2}\right) - 4\cot^{-1}(\sqrt{3})$

(v) $3\operatorname{cosec}^{-1}(1) + \sec^{-1}(2) - 5\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) + 7\cot^{-1}\left(\frac{1}{\sqrt{3}}\right)$

UNIT – II
MATRICES & DETERMINANTS
Multiple Choice Questions(1 Mark)

1 If order of matrix A is 2×3 and order of matrix B is 3×5 then order of matrix $B'A'$ is :

(a)5×2 (b) 2×5 (c) 5×3 (d) 3×2

2 If $\begin{vmatrix} x & 1 \\ 1 & x \end{vmatrix} = \begin{vmatrix} 2 & 0 \\ 8 & 4 \end{vmatrix}$ then value of x is :

(a)3 (b)2 (c)4 (d)8

3 If $\begin{bmatrix} 2x+y & 0 \\ 5 & x \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 5 & 3 \end{bmatrix}$, then y is equal to:-

(a)1 (b)3 (c)2 (d)-1

4 If A + B = C where B and C are matrices of order 5×5 then order of A is :-

(a) 5×5 (b) 5×3 (c) 3×5 (d) 3×3

5 If A B = C where B and C are matrices of order 2×5 and 5×5 respectively then order of A is :-

(a) 5×5 (b) 5×2 (c) 2×5 (d) 2×2

6 If order of matrix A is 2×3 and order of matrix B is 3×5 then order of matrix AB is :

(a)5×2 (b) 2×5 (c) 5×3 (d) 3×2

7 If A is a square matrix of order 3×3 and $|A| = 7$ then determinant of the matrix $2A$ is

(a)14 (b)28 (c)42 (d)56

8 If A is a square matrix of order 4×4 and $|A| = 3$ then $|Adj.(A)|$ is

(a)27 (b)81 (c)9 (d)3

9 If $A = \begin{bmatrix} 2 & 5 \\ 1 & -2 \end{bmatrix}$ then $|A|$ is

(a)-9 (b)9 (c)1 (d)-1

10 If A is a matrix of order of 3×3 and $|A| = 3$ then $|Adj(A)|$ is

(a)81 (b)9 (c)27 (d)3

Fill-ups(1 Mark)

1) If $A = [a_{ij}]_{2\times3}$ such that $a_{ij} = i + j$ then $a_{11} =$__________

2) If $|A| = 5$ where A is a matrix of order 3×3 then $|adj.(A)| =$__________

3) If matrix $A = \begin{bmatrix} 2 & 3 \\ 1 & 5 \end{bmatrix}$ then $|A| =$__________

4) If order of matrix A is 3×4 then order of $A' =$_________________

5) If for a matrix , $A' = A$ holds then A is _________________ matrix.

6) If for a matrix , $A' = -A$ holds then A is _________________ matrix.

7) If for any two matrices A and B, $AB = BA = I$ then these matrices are __________ of each other.

8) _________ matrix is symmetric as well as skew-symmetric.

9) If order of matrix A is 3×4 and order of matrix B is 4×7 then order of AB is ____________.

10) If order of matrix A is 4×5 then number of elements in A are ____________

2 Marks Questions

1. If $A = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$, then verify $A^2 - 7A - 2I = 0$.

2. If $A = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$ show that $A^2 - 5A - 14I = 0$.

3. If $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$ and $f(x) = x^2 - 4x + 1$ then find $f(A)$.

4. If $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$and $f(x) = x^2 - 2x - 3$ then find $f(A)$.

5. If $A = \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix}$ and $A^2 - 8A = kI$ then find k.

6. If $A = \begin{bmatrix} 0 & 3 \\ -7 & 5 \end{bmatrix}$ then find k so that $kA^2 = 5A - 21I$.

7. If $X = \begin{bmatrix} 3 & 4 \\ 2 & -1 \end{bmatrix}$ and $2X - Y = \begin{bmatrix} 5 & 10 \\ 3 & -5 \end{bmatrix}$ then find the matrix Y.

8. If $X - 2Y = \begin{bmatrix} 5 & 1 \\ 2 & 0 \end{bmatrix}$ and $2X - Y = \begin{bmatrix} 4 & 9 \\ 1 & -3 \end{bmatrix}$ then find the matrices X and Y.

9. Verify $(AB)' = B'A'$ for the following matrices :

   (i) $A = \begin{bmatrix} 1 \\ 3 \\ 6 \end{bmatrix}$, $B = [2 \quad 4 \quad 5]$

   (ii) $A = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$, $B = [-2 \quad -1 \quad -4]$

   (iii) $A = \begin{bmatrix} 2 & 3 \\ 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 4 \\ 2 & 1 \end{bmatrix}$

   (iv) $A = \begin{bmatrix} -1 & 3 & 0 \\ -7 & 2 & 8 \end{bmatrix}$, $B = \begin{bmatrix} -5 & 0 \\ 0 & 3 \\ 1 & -8 \end{bmatrix}$

   (v) $A = \begin{bmatrix} 2 & 1 & 3 \\ 4 & 1 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 1 & -1 \\ 0 & 2 \\ 5 & 0 \end{bmatrix}$

10. Using determinants, show that following points are collinear :

    (i) $(11, 7), (5, 5)$and $(-1, 3)$

    (ii) $(3, 8), (-4, 2)$and $(10, 14)$

    (iii) $(-2, 5), (-6, -7)$and $(-5, -4)$

11. Find the value of x if $(3, -2), (x, 2)$and $(8, 8)$ are collinear points.

12. Using determinants, find the value of k if the area of the triangle formed by the points $(-3, 6), (-4, 4)$and $(k, -2)$ is 12 sq. units.

13. If the area of triangle is 35 sq. units with vertices $(2, -6), (5, 4)$and $(k, 4)$ then find the value of k.

14. Find the equation of the line passing from $(3, 2)$ and $(-4, -7)$ using determinants.

6Marks Questions

1. Solve the following system of linear equations by matrix method :

(i) $x - y + 2z = 7,\ 3x + 4y - 5z = -5,\ 2x - y + 3z = 12$

(ii) $x + y + z = 6,\ y + 3z = 11,\ x - 2y + z = 0$

(iii) $3x + y + z = 10,\ 2x - y - z = 0,\ x - y + 2z = 1$

(iv) $2x + 3y + 3z = 5,\ x - 2y + z = -4,\ 3x - y - 2z = 3$

(v) $2x + 3y + 3z = 5,\ x - 2y + z = -4,\ 3x - y - 2z = 3$

(vi) $x - y + 2z = 2,\ 3x + 4y - 5z = 2,\ 2x - y + 3z = 4$

(vii) $x + y - z = 3,\ 2x + 3y + z = 10,\ 3x - y - 7z = 1$

(viii) $x + y + z = 3\ ,\ 5x - y - z = 3\ ,\ 3x + 2y - 4z = 1$

(ix) $2x + 3y + 3z = 5,\ x - 2y + z = -4,\ 3x - y - 2z = 3$

(x) $5x + y - z = -6\ ,\ 2x - 3y + 4z = 3\ ,\ 7x + y - 3z = -12$

2. Express the following matrices as a sum of a symmetric matrix and a skew-symmetric matrix :

(i) $\begin{bmatrix} 2 & 0 & 3 \\ -1 & 4 & 8 \\ 7 & 2 & 9 \end{bmatrix}$ **(ii)** $\begin{bmatrix} 3 & 6 & 2 \\ 0 & 7 & 8 \\ 5 & 1 & 9 \end{bmatrix}$

(iii). $\begin{bmatrix} 5 & 1 & 2 \\ -2 & 3 & 0 \\ 6 & 3 & 7 \end{bmatrix}$ **(iv)** $\begin{bmatrix} 2 & 5 & 8 \\ -3 & 6 & 0 \\ 5 & 2 & 1 \end{bmatrix}$

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UNIT – III
Continuity & Differentiability
Multiple Choice Questions (1 Marks)

1 If $f(x) = \begin{cases} kx+1, & x \le 5 \\ 3x-5, & x > 5 \end{cases}$ is continuous then value of k is :

(a)$\frac{9}{5}$ (b) $\frac{5}{9}$ (c) $\frac{5}{3}$ (d) $\frac{3}{5}$

2 If $f(x) = \begin{cases} kx^2, & x \le 2 \\ 3, & x > 2 \end{cases}$ is continuous then value of k is :

(a)$\frac{2}{3}$ (b) $\frac{4}{3}$ (c) $\frac{3}{2}$ (d) $\frac{3}{4}$

3 If $f(x) = \begin{cases} mx-1, & x \le 5 \\ 3x-5, & x > 5 \end{cases}$ is continuous then value of m is :

(a)$\frac{11}{5}$ (b) $\frac{5}{11}$ (c) $\frac{5}{3}$ (d) $\frac{3}{5}$

4 If $f(x) = \begin{cases} mx^2, & x \le 5 \\ 6x-5, & x > 5 \end{cases}$ is continuous then value of m is :

(a)- 1 (b)4 (c)3 (d)1

5 If $f(x) = \begin{cases} \frac{\sin 2x}{3x}, & x \ne 0 \\ m-1, & x = 0 \end{cases}$ is continuous then value of m is :

(a)2/3 (b)3/2 (c)3/5 (d)5/3

6 If $f(x) = \begin{cases} kx+1, & x \le 5 \\ 3x+5, & x > 5 \end{cases}$ is continuous then value of k is :

(a)$\frac{19}{5}$ (b) $\frac{5}{9}$ (c) $\frac{5}{3}$ (d) $\frac{3}{5}$

7 If $f(x) = \begin{cases} kx-1, & x \le 5 \\ 3x+5, & x > 5 \end{cases}$ is continuous then value of k is :

(a)$\frac{21}{5}$ (b) $\frac{5}{19}$ (c) $\frac{5}{21}$ (d) $\frac{19}{5}$

8 If $f(x) = \begin{cases} \frac{\sin 7x}{3x}, & x \ne 0 \\ m, & x = 0 \end{cases}$ is continuous at $x = 0$ then value of m is

(a)$\frac{3}{7}$ (b)$\frac{4}{7}$ (c)$\frac{7}{4}$ (d)$\frac{7}{3}$

9 If $y = \log\left[x + \sqrt{x^2+1}\right]$ then $\frac{dy}{dx}$ is

(a)$\sqrt{x^2+1}$ (b)$\frac{1}{\sqrt{x^2+1}}$ (c)$\frac{x}{\sqrt{x^2+1}}$ (d)$\frac{1}{x+\sqrt{x^2+1}}$

10 If $f(x) = \begin{cases} \frac{x^3-8}{x-2}, & x \ne 2 \\ k, & x = 2 \end{cases}$ is continuous at $x = 2$ then value of k is

(a)8 (b)2 (c)6 (d)12

11 $\frac{d}{dx}\{\tan^{-1}(e^x)\}$ is equal to :

(a)$e^x \tan^{-1} e^x$ (b) $\frac{e^x}{1+e^{2x}}$ (c) 0 (d)$e^x \sec^{-1} x$

12 If $y = \sin x$ then at $x = \frac{\pi}{2}$, y_2 is equal to :

(a)-1 (b) 1 (c) 0 (d) $\frac{1}{2}$

13 If $x = 2at, y = at^2$ then $\frac{dy}{dx}$ is equal to:

(a)2 (b)$2a$ (c)$2at$ (d)t

14 If $y = \cos^{-1}(e^x)$ then $\frac{dy}{dx}$ is equal to:

(a)$e^x \sin^{-1}(e^x)$ (b) $e^x \cos^{-1}(e^x)$ (c)$\frac{-e^x}{\sqrt{1-e^{2x}}}$ (d)$\frac{e^x}{\sqrt{1-e^{2x}}}$

15 If $y = \sin^{-1}(e^x)$ then $\frac{dy}{dx}$ is equal to:

(a)$e^x \sin^{-1}(e^x)$ (b) $e^x \cos^{-1}(e^x)$ (c)$\frac{-e^x}{\sqrt{1-e^{2x}}}$ (d)$\frac{e^x}{\sqrt{1-e^{2x}}}$

16 $\frac{d}{dx}\{\cot^{-1}(e^x)\}$ is equal to :

(a)$e^x \tan^{-1} e^x$ (b) $\frac{e^x}{1+e^{2x}}$ (c) $\frac{-e^x}{1+e^{2x}}$ (d)$e^x \sec^{-1} x$

17 If $y = x^2$ then $y_1(5)$ is equal to :

(a)10 (b)25 (c)32 (d) 0

18 If $y = \log(\sin x)$ then at $x = \frac{\pi}{4}, \frac{dy}{dx}$ is

(a)0 (b)-1 (c)1 (d)$\sqrt{2}$

19 If $y = e^{\log x}$ then $\frac{dy}{dx}$ is

(a)$\log x - x$ (b)$xe^{\log x}$ (c)1 (d)$e^{\log x} \log x$

20 If $y = \log(\sec x)$ then $\frac{dy}{dx}$ at $x = \frac{\pi}{4}$ is

(a)1 (b)-1 (c)0 (d)10

True/False (1 Marks)

1) If $y = 10x$ then $\frac{dy}{dx} = 0$.
2) If $y = 500$ then $\frac{dy}{dx} = 0$
3) If $y = \tan x$ then $\frac{dy}{dx} = \sin x$
4) If $y = \cot x$ then $\frac{dy}{dx} = \log(\cos x)$
5) If $y = \tan 2x$ then $\frac{dy}{dx} = 2\sec^2 2x$
6) Trigonometric functions are differentiable functions in their respective domains.
7) The composition of two continuous functions is continuous.
8) Every differentiable function is a continuous function.
9) Logarithmic differentiation is essential for the function f when $f(x) = \left(p(x)\right)^{q(x)}$
10) Exponential function is not a continuous function.
11) Every continuous function is differentiable.
12) A function is called continuous at a point if its limit exists at that point.

2 Marks Questions

1. Find $\frac{dy}{dx}$ for the following parametric functions :

(i) $x = a\cos^2\theta$, $y = b\sin^2\theta$

(ii) $x = a(\theta - \sin\theta)$, $y = b(1 + \cos\theta)$

(iii) $x = a(\theta + \sin\theta)$, $y = b(1 + \cos\theta)$

(iv) $x^2 + y^2 + 2xy = 23$

(v) $y = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$

(vi) $y = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$

(vii) $x^3 + 3x^2y + 3xy^2 + y^3 = 81$

4Marks Questions

1) If $x = 2\cos\theta - \cos 2\theta$, $y = 2\sin\theta - \sin 2\theta$ then find $\frac{dy}{dx}$ at $\theta = \frac{\pi}{2}$.

2) If $x = \frac{1-t^2}{1+t^2}$, $y = \frac{2t}{1+t^2}$ then prove that $\frac{dy}{dx} + \frac{x}{y} = 0$.

3) Differentiate the following w.r.t. x :

(i) $x^{\sin x} + (\sin x)^x$ (ii) $x^{\log x} + (\log x)^x$ (iii) $x^{\tan x} + (\tan x)^x$

(iv) $x^{\cos x} + (\sin x)^{\tan x}$ (v) $x^x + (\sin x)^x$ (vi) $x^{\sin^{-1} x} + (\sin^{-1} x)^x$

4) Solve the following :

(i) If $(\sin x)^y = (\sin y)^x$, find $\frac{dy}{dx}$.

(ii) If $(\sin x)^y = (\cos y)^x$, find $\frac{dy}{dx}$.

(iii) If $y = x^y$ show that $\frac{dy}{dx} = \frac{y^2}{x(1-y\log x)}$.

(iv) If $x^y + y^x = \log a$, find $\frac{dy}{dx}$.

5) If $y = \sin^{-1} x$ then show that $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 0$.

6) If $y = (\sin^{-1} x)^2$ then prove that $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 2$.

7) If $y = (\tan^{-1} x)^2$ then show that $(1+x^2)^2 y_2 + 2x(1+x^2)y_1 - 2 = 0$.

8) If $y = \log\left(x + \sqrt{x^2+1}\right)$ then show that $(x^2+1)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = 0$.

9) If $y = e^{m\sin^{-1} x}$ then show that $(1-x^2)y_2 - xy_1 - m^2 y = 0$.

10) If $y = e^{m\tan^{-1} x}$ prove that

(i) $(1+x^2)\frac{d^2y}{dx^2} + (2x-m)\frac{dy}{dx} = 0$ (ii) $(1+x^2)^2 y_2 + 2x(1+x^2)y_1 - m^2 y = 0$.

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UNIT – III

Applications of Derivatives

Multiple Choice Questions (1 Marks)

1 Rate of change of perimeter of a square with respect to its side is :
(a)2 (b)1 (c)4 (d)3

2 Radius of a circle is increasing at the rate of $2\ m/s$. Rate of change of its circumference is :
(a)$4\pi\ m/s$ (b)$2\ m/s$ (c)$2\pi\ m/s$ (d)$4\ m/s$

3 Radius of a sphere is increasing at the rate of $5\ m/s$. Rate of change of its surface area, when radius is $4\ m$, is
(a)$120\pi\ m^2/s$ (b) $160\pi\ m^2/s$ (c) $32\pi\ m^2/s$ (d) $80\pi\ m^2/s$

4 $f(x) = \sin x$ is strictly decreasing in the interval :
(a)$\left(\frac{\pi}{2}, \pi\right)$ (b)$\left(\pi, \frac{3\pi}{2}\right)$ (c)$\left(0, \frac{\pi}{2}\right)$ (d)$\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$

5 $f(x) = \cos x$ is strictly increasing in the interval :
(a)$\left(\frac{\pi}{2}, \pi\right)$ (b)$\left(\pi, \frac{3\pi}{2}\right)$ (c)$\left(0, \frac{\pi}{2}\right)$ (d)$\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$

6 $f(x) = x^2$ strictly increases on :
(a)$(0, \infty)$ (b)$(-\infty, 0)$ (c)$(-7, -3)$ (d)$(-\infty, -3)$

7 If f is differentiable at critical points then the value of derivative of f at critical point is :
(a)1 (b)-1 (c)0 (d)2

8 On the curve $y = f(x)$ if $f'(a) = 0$ then $x = a$ is called a
(a)Practical point on the curve (b)Critical point on the curve
(c)Maximum point on the curve (d)Minimum point on the curve

9 On the curve $y = f(x)$ if $f'(a) = 0$ and $f''(a) < 0$ then $x = a$ is point of
(a)Maxima (b)Minima (c)Inflexion (d)infinity

10 On the curve $y = f(x)$ if $f'(a) = 0$ and $f''(a) > 0$ then $x = a$ is point of
(a)Maxima (b)Minima (c)Inflexion (d)infinity

True/False

1) Function f decreases where $f'(x) > 0$.
2) Function f decreases where $f'(x) < 0$.
3) $f(x) = \sin x$ is strictly decreasing function in $\left[0, \frac{\pi}{2}\right]$.
4) The value of function f is maximum at a if $f'(a) = 0$ and $f''(a) < 0$.
5) Logarithmic function $f(x) = \log x$ is a strictly increasing function.
6) Velocity of a moving particle cannot be expressed as derivative of displacement function of the particle.
7) The value of function f is maximum or minimum at a if $f'(a) = 0$.
8) The value of function f is minimum at a if $f'(a) = 0$ and $f''(a) < 0$.
9) When $f'(a) = 0$ then $x = a$ is called a critical point on the curve $y = f(x)$
10) If a given cylindrical bucket is being filled with water with a given rate then we can evaluate the rate of change of the volume of water cylinder inside the bucket.

2 Marks Questions

1. The volume of spherical balloon is increasing at the rate of 25 c.c./s . Find the rate of change of its surface area at the instant when its radius is 5 cm.
2. The side of square sheet is increasing at the rate of 3 cm/s. At what rate is the area increasing when the side is 10 cm long?
3. The side of square sheet is increasing at the rate of 5 cm/s. At what rate is the perimeter increasing when the side is 7 cm long?
4. The radius of spherical soap bubble is increasing at the rate of 0.2 cm/s. Find the rate of change of its volume when its radius is 4 cm.

5. The radius of spherical soap bubble is increasing at the rate of 0.8 cm/s. Find the rate of change of its surface area when the radius is 5 cm.
6. The edge of a cube is decreasing at the rate of 2 cm/s . Find the rate of change of its volume when the length of edge of the cube is 5 cm.
7. The edge of a cube is decreasing at the rate of 2 cm/s . Find the rate of change of its surface area when the length of edge is 6 cm.
8. Determine the intervals in which the following functions are increasing or decreasing :

(a) $f(x) = x^3 + 2x^2 - 1$ (b) $f(x) = 30 - 24x + 15x^2 - 2x^3$ (c) $f(x) = 20 - 12x + 9x^2 - 2x^3$
(d) $f(x) = 17 - 18x + 12x^2 - 2x^3$ (e) $f(x) = 20 - 9x + 6x^2 - x^3$ (f) $f(x) = 6 + 12x + 3x^2 - 2x^3$
(g) $f(x) = 2x^3 - 15x^2 + 36x + 1$ (h) $f(x) = x^3 - 6x^2 + 9x + 8$ (i) $f(x) = 2x^3 - 12x^2 + 18x + 5$

6 Marks Questions

1. Find the volume of the biggest right circular cone which is inscribed in a sphere of radius $9cm$.
2. Prove that the height of a right circular cylinder of maximum volume, which is inscribed in a sphere of radius R, is $\frac{2R}{\sqrt{3}}$.
3. Show that of all the rectangles inscribed in a given fixed circle, the square has the maximum area.
4. A wire of length 25 m is to be cut into two pieces. One of the pieces is to be made into a square and the other into a circle. What could be the lengths of the two pieces so that the combined area of the square and circle is minimum?
5. A wire of length 36 m is to be cut into two pieces. One of the pieces is to be made into a square and the other into an equilateral triangle. What could be the lengths of the two pieces so that the combined area of the square and equilateral triangle is minimum?
6. Prove that the perimeter of a right angled triangle of given hypotenuse equal to 5 cm is maximum when the triangle is isosceles.
7. Of all rectangles with perimeter 40 cm/100 cm/80 cm find the one having maximum area. Also find the area.
8. Find the volume of the largest cylinder that can be inscribed in a sphere of radius R.
9. Find the volume of largest cone that can be inscribed in a sphere of radius R.
10. Show that height of the cylinder of maximum volume that can be inscribed in a sphere of 30 cm is $\frac{60}{\sqrt{3}}$ cm.
11. A window is in the form of rectangle surmounted by a semi-circle opening. If the perimeter of window is 10 cm/20 cm/ 30 cm, find the dimensions of the window so as to admit maximum possible light through the whole opening.
12. Show that the height of a closed cylinder of given volume and least surface area is equal to its diameter.
13. Find two positive numbers whose sum is 16 and the sum of whose cubes is maximum.
14. Prove that the volume of the largest cone that can be inscribed in a sphere of radius R is $\frac{8}{27}$ of the volume of the sphere.
15. A square piece of tin of side 18cm is to be made into a box without top, by cutting off small squares from each corner and folding up the flaps to form the box. What should be the side of the small square to be cut off so that the volume of the box is the maximum possible.

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INTEGRALS

Fill in the Blanks(1 Marks)

1) $\int_0^3 dx =$__________
2) $\int \sec^2 x\, dx =$__________
3) $\int_0^3 3x^2 dx =$__________
4) $\int_0^5 2x dx =$__________
5) Integration is ______________ process of differentiation.
6) $\int_{-a}^{a} f(x)\, dx = 0$ if f is __________________function.
7) $\int \frac{dx}{1+x^2} =$_______________
8) $\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx =$_________________
9) $\int \frac{1}{x+3} dx =$ ____________
10) $\int \sec x \tan x\, dx =$ ___________

Multiple Choice Questions(1 Marks)

1 $\int_0^{\pi/2} \frac{\sin^{1/2} x}{\sin^{1/2} x+\cos^{1/2} x} dx$ is equal to :

(a)0 (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{4}$

2 $\int_0^{\pi/2} \frac{\sin^{3/2} x}{\sin^{3/2} x+\cos^{3/2} x} dx$ is equal to :

(a)0 (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{4}$

3 $\int \frac{dx}{2x+3}$ equals:

(a)$\log|2x+3| + c$ (b)$\log|2x-3| + c$ (c)$\frac{\log|2x+3|}{3} + c$ (d)$\frac{\log|2x+3|}{2} + c$

4 $\int \frac{dx}{2x-5}$ equals:

(a)$\log|2x-5| + c$ (b)$\log|2x+5| + c$ (c)$\frac{\log|2x-5|}{5} + c$ (d)$\frac{\log|2x-5|}{2} + c$

5 $\int_{-1}^{1} x^3 \cos x\, dx$ equals:

(a)0 (b)1/4 (c)π (d)none of these

6 $\int_0^{\pi/2} \frac{\sin^{1/2} x}{\sin^{1/2} x+\cos^{1/2} x} dx$ is equal to :

(a)0 (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{4}$

7 $\int_{-2}^{2} x^3 dx$ is equal to :

(a)0 (b)4 (c)16/3 (d) $\frac{\pi}{4}$

8 $\int_{-1}^{1} x \sin^2 x\, dx$ is equal to

(a)0 (b)$\frac{1}{2}$ (c)$\frac{1}{3}$ (d)-1

9 $\int_0^1 \frac{dx}{1+x^2}$ is

(a)$\frac{\pi}{2}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{6}$

10 $\int_{\pi/6}^{\pi/3} \frac{\cos^4 x}{\sin^4 x+\cos^4 x} dx$ is equal to

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{12}$ (d) $\frac{\pi}{2}$

2 Marks Questions

1. **Evaluate the following :**

(a) $\int \frac{(x-4)^3}{x^2} dx$ (b) $\int \frac{dx}{1-\sin x}$ (c) $\int \frac{dx}{1+\cos x}$ (d) $\int \frac{dx}{1+\sin x}$

(e) $\int \frac{dx}{1-\cos x}$ (f) $\int \frac{e^x-1}{e^x+1} dx$ (g) $\int \frac{(\tan^{-1} x)^2}{1+x^2} dx$ (h) $\int \frac{\sec^2(2\tan^{-1} x)}{1+x^2} dx$

(i) $\int \frac{\sin(\tan^{-1} x)}{1+x^2} dx$ (j) $\int \frac{dx}{x^2+8x-9}$ (k) $\int \frac{dx}{\sqrt{x^2-5x+7}}$ (l) $\int \frac{dx}{\sqrt{x^2+4x+7}}$

(m) $\int \frac{dx}{x^2+6x+5}$ (n) $\int \frac{dx}{x^2-6x+18}$ (o) $\int x\sqrt{x+2}dx$ (p) $\int \frac{3-2\sin x}{\cos^2 x} dx$

2. **Compute the following :**

(a) $\int \frac{\log x}{x} dx$ (b) $\int \frac{e^{\tan^{-1} x}}{1+x^2} dx$ (c) $\int \frac{2x}{1+x^2} dx$ (d) $\int \frac{x^2}{1+x^3} dx$

(e) $\int \frac{6x-8}{3x^2-8x+5} dx$ (f) $\int \frac{2x+9}{x^2+9x+20} dx$ (g) $\int e^x \left(\tan^{-1} x + \frac{1}{1+x^2}\right) dx$

4 Marks Questions

3. **Integrate the following :**

(a) $\sin^2 x \cos^3 x$ (b) $\cos^2 x \sin^3 x$ (c) $\frac{1}{1-\cot x}$

(d) $\frac{1}{1+\cot x}$ (e) $\frac{1}{1-\tan x}$ (f) $\frac{1}{1+\tan x}$

4. **Evaluate the following integrals :**

(a) $\int \frac{1-\tan x}{1+\tan x} dx$ (b) $\int \frac{1+\tan x}{1-\tan x} dx$

5. **Integrate the following functions:**

(a) $x\sec^2 x$ (b) $x^2 e^x$ (c) $x\cos 3x$ (d) $x\sin x$

7. **Integrate the following functions :**

(a) $e^x \sin 2x$ (b) $e^{3x} \cos 5x$ (c) $e^x(\cot x + \log\sin x)$

8. **Integrate the following functions :**

(a) $\frac{1}{(x+1)(x+2)(x+3)}$ (b) $\frac{1}{x(x-1)(x-2)}$ (c) $\frac{1}{x^3-1}$ (d) $\frac{1}{(1-x)(1+x^2)}$

(e) $\frac{x}{(x-2)(x^2+4)}$ (f) $\frac{1}{x(x^2+2)}$

9. **Integrate the following functions :**

   (a) $\frac{2x}{\sqrt{(x+1)(x-2)}}$ (b) $\frac{4x+5}{\sqrt{x^2+x-3}}$ (c) $\frac{3x+5}{\sqrt{x^2-8x+7}}$

12 **Evaluate the following integrals :**

   (a) $\int_0^{\pi/2} \frac{\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}}\,dx$ (b) $\int_0^{\pi/2} \frac{1}{1+\sqrt{\cot x}}\,dx$

   (c) $\int_0^{\pi/2} \frac{\sqrt{\cot x}}{\sqrt{\tan x}+\sqrt{\cot x}}\,dx$ (e) $\int_0^1 |x-5|\,dx$

   (f) $\int_{-6}^{6} |x+2|\,dx$ (h) $\int_0^{\pi/2} \log(\cos x)\,dx$

6 Marks Questions

Evaluate the following :

1. $\int \frac{x^2+1}{x^4+1}\,dx$
2. $\int \frac{x^2}{x^4+1}\,dx$
3. $\int \frac{1}{x^4+1}\,dx$
4. $\int_0^{\pi/2} \log \cos x\,dx$
5. $\int \frac{2x}{(x^2+1)(x^2+4)}\,dx$
6. $\int \frac{1}{x^3-1}\,dx$
7. $\int \left(\sqrt{\cot x}+\sqrt{\tan x}\right)dx$
8. $\int_0^{\pi} \frac{x\,dx}{a^2\cos^2 x+b^2\sin^2 x}$

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APPLICATIONS OF INTEGRALS

2 Marks Questions

1. **Using integration, find the area of the circle :**

   (i) $x^2 + y^2 = 4$ **(ii)** $x^2 + y^2 = 9$

   (iii) $\frac{x^2}{9} + \frac{y^2}{4} = 1$ **(iv)** $\frac{x^2}{9} + \frac{y^2}{25} = 1$

   (v) $\frac{x^2}{16} + \frac{y^2}{36} = 1$ **(vi)** $\frac{x^2}{25} + \frac{y^2}{16} = 1$

2. **Using integration, find the area bounded between parabola $x^2 = 4y$ and line $y = 4$.**
3. **Find the area of the region bounded by $x^2 = 4y, y = 2, y = 4$ and y – axis in first quadrant.**
4. **Draw a rough sketch to indicate the region bounded between the curve $y^2 = 4x, x = 3$. Also find the area of this region.**
5. **Find the area of the region bounded by the curves $y^2 = 8x$, $x = 1$, $x = 5$ in the first quadrant.**
6. **Find the area of the region bounded by the parabola $x^2 = 9y$ and lines $x = 1$, $x = 4$ and x –axis.**
7. **Find the area bounded by the lines $x = 2, x = 7, y = 9$ and x –axis.**
8. **Find the area bounded by the lines $y = x$, $x = 5$ and $x -$ axis.**

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DIFFERENTIAL EQUATIONS

Multiple Choice Questions(1 Marks)

1 Integrating factor of differential equation $\frac{dy}{dx}-\frac{y}{x}=2x$ is :

(a) $\frac{1}{x}$ (b) x (c) $\frac{1}{x^2}$ (d) 1

2 Order of differential equation $\frac{d^2y}{dx^2}-\left(\frac{dy}{dx}\right)^3+y=0$ is :

(a) 3 (b) 2 (c) 0 (d) 1

3 Which of the following differential equations is a homogeneous type of differential equation :

(a)$(4x+6y+5)dy-(3y+2x+4)dx=0$ (b)$xydx-(x^3+y^3)dy=0$

(c)$(x^3+2y^2)dx+2xydy=0$ (d)$y^2dx+(x^2-xy-y^2)dy=0$

4 Integrating factor of differential equation $\frac{dy}{dx}+\frac{y}{x}=2x$ is:

(a)$\frac{1}{x}$ (b)x^2 (c) $\frac{1}{x^2}$ (d)x

5 Integrating factor of differential equation $\frac{dy}{dx}+\frac{2y}{x}=2x$ is:

(a)$\frac{1}{x}$ (b)x^2 (c) $\frac{1}{x^2}$ (d)x

6 Integrating factor of differential equation $\frac{dy}{dx}+y\sec x=2x$ is :

(a)$\sec x+\tan x$ (b) $\sec x\tan x$ (c)$e^{\sec x}$ (d) $e^{\sec x+\tan x}$

7 Integrating factor of differential equation $\frac{dy}{dx}+y=2x$ is :

(a) $\frac{1}{x}$ (b) x (c)e^x (d) e^{-x}

8 Order of differential equation $\frac{d^3y}{dx^3}-4\left(\frac{d^2y}{dx^2}\right)^4+y=0$ is

(a)3 (b)4 (c)1 (d)0

9 The number of arbitrary constants in the general solution of a differential equation of second order are

(a)1 (b)2 (c)3 (d)0

10 Degree of the differential equation $\frac{d^3y}{dx^3}-\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^4+y=0$ is

(a)1 (b)2 (c)3 (d)4

Fill Ups(1 Marks)

1) Order of the differential equation $\frac{d^2y}{dx^2}-\left(\frac{dy}{dx}\right)^3+y=0$ is _________

2) Degree of the differential equation $\frac{d^2y}{dx^2}-\left(\frac{dy}{dx}\right)^3+y=0$ is _________

3) Integrating factor of differential equation $\frac{dy}{dx}+xy=\sin x$ is _________

4) Order and degree (if defined) of a differential equation are always _____________ integers.

5) Integrating factor of $\frac{dx}{dy}+Px=Q$ is ____________

6) $(x+y)dy-(x-2y)dx=0$ is a ________________differential equation.

7) _________ substitution is applied to solve a homogeneous differential equation.

8) There are ________number of arbitrary constants in the general solution of differential equation of order 3.

9) $y=$ _____ substitution is used in the $\frac{dy}{dx}=f\left(\frac{y}{x}\right)$ type of homogeneous differential equation.

10) After correct substitution, a homogeneous differential equation becomes ______________ type of differential equation.

4 Marks Questions

Solve the following differential equations :

1) $\frac{dy}{dx} = \log x$
2) $\frac{dy}{dx} + \frac{1+y^2}{y} = 0$
3) $\frac{dy}{dx} = \sin^2 y$
4) $\frac{dy}{dx} = e^y \sin x$
5) $\log \frac{dy}{dx} = ax + by$
6) $x^2(y+1)dx + y^2(x-1)dy = 0$
7) $\sec^2 x \tan y \, dx - \sec^2 y \tan x \, dy = 0$
8) $xdy + ydx = xydx \,;\, y(1) = 1$
9) $x(xdy - ydx) = ydx \,;\, y(1) = 1$
10) $\frac{dy}{dx} = y \tan x \,;\, y(0) = 1$
11) $\frac{dy}{dx} = y \sin 2x \,;\, y(0) = 1$
12) $(x^2 + xy)dy + (3xy + y^2)dx = 0$
13) $(y^2 - x^2)dy - 3xydx = 0$
14) $2xydx + (x^2 + 2y^2)dy = 0$
15) $x^2 dy - (x^2 + xy + y^2)dx = 0$
16) $\cos\left(\frac{dy}{dx}\right) = \frac{1}{9} \,;\, y(0) = 2$
17) $(x^2 + y^2)dx + 2xydy = 0$
18) $(x^2 - 2y^2)dx + xydy = 0$
19) $\frac{dy}{dx} + \frac{y}{x} = e^x$
20) $\frac{dy}{dx} - 4y = e^{2x}$
21) $x\frac{dy}{dx} + y = x^3$
22) $\frac{dy}{dx} + 2y = \sin 5x$
23) $\frac{dy}{dx} + 3y = \cos 2x$
24) $x\frac{dy}{dx} + y = x \log x$
25) $(1 + x^2)\frac{dy}{dx} + y = \tan^{-1} x$
26) $\frac{dy}{dx} = 2x + y \,;\, y(0) = 0$
27) $\frac{dy}{dx} = 4x + y \;;\; y(0) = 1$
28) $x\frac{dy}{dx} + y = x^3 \;;\; y(2) = 1$
29) $xy' - y = \log x \;;\; y(1) = 0$
30) $x \log x \frac{dy}{dx} + y = \frac{2}{x} \log x$
31) $\frac{dy}{dx} = \frac{1-\cos x}{1+\cos x}$
32) $(1 + x^2)dy + 2xydx = \cot x \, dx$
33) $x\frac{dy}{dx} + 2y = x^2 \log x$
34) $x^2 dy - (3x^2 + xy + y^2)dx = 0 \,;\, y(1) = 1$.

Vector Algebra

True/False(1 Mark)

1) Scalar product of two perpendicular vectors is zero.
2) Vector product of two collinear vectors is zero.
3) $|\vec{a}.\vec{b}| \leq |\vec{a}||\vec{b}|$ is triangle inequality.
4) $|\vec{a} + \vec{b}| \leq |\vec{a}| + |\vec{b}|$ is Cauchy-Schwartz inequality.
5) $\hat{\imath} \times \hat{\imath} = \hat{\jmath} \times \hat{\jmath} = \hat{k} \times \hat{k} = \vec{0}$.
6) $\hat{\imath}.\hat{\imath} = \hat{\jmath}.\hat{\jmath} = \hat{k}.\hat{k} = 0$
7) $\vec{a} \times \vec{b}$ is parallel to both the vectors $\vec{a}$ and $\vec{b}$.
8) Vectors $3\hat{\imath} + 2\hat{\jmath} - \hat{k}$ is parallel to the vector $\hat{\imath} + 6\hat{\jmath} - 3\hat{k}$.
9) Area of a parallelogram can be calculated by using vector product of two vectors.
10) Area of a triangle cannot be calculated by using vector product of two vectors.

Multiple Choice Questions(1 Mark)

1 If $\vec{a}.\vec{b} = |\vec{a} \times \vec{b}|$ then angle between vector $\vec{a}$ and vector $\vec{b}$ is :

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{3}$

2 Magnitude of the vector $\frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} + \frac{1}{\sqrt{3}}\hat{k}$ is :

(a) -1 (b) 1 (c) 0 (d) $\frac{1}{3}$

3 If $\sqrt{3}\,\vec{a}.\vec{b} = |\vec{a} \times \vec{b}|$ then angle between vector $\vec{a}$ and vector $\vec{b}$ is :

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{3}$

4 If $\vec{a}.\vec{b} = \sqrt{3}|\vec{a} \times \vec{b}|$ then angle between vector $\vec{a}$ and vector $\vec{b}$ is :

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{3}$

5 If $\vec{a}.\vec{b} = 0$ then angle between vector $\vec{a}$ and vector $\vec{b}$ is :

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{3}$

6 Name of the inequality $|\vec{a}.\vec{b}| \leq |\vec{a}||\vec{b}|$ is :

(a)Cauchy-Schwartz Inequality (b)Triangle Inequality

(c)Rolle's Theorem (d)Lagrange's Mean Value Theorem

7 Magnitude of vector $\vec{a} = 3\hat{i} + \hat{j} + \hat{k}$ is :

(a) 3 (b) $\sqrt{10}$ (c) $\sqrt{11}$ (d) $\sqrt{12}$

8 Projection of $\vec{a} = 3\hat{i} + \hat{j} + \hat{k}$ on $\vec{b} = \hat{i} - 2\hat{j} - \hat{k}$ is :

(a)$\frac{2}{\sqrt{6}}$ (b)0 (c)$\frac{1}{\sqrt{6}}$ (d)$\sqrt{6}$

9 If $\vec{a}$ is a non-zero vector then $|\vec{a} \times \vec{a}|$ is equal to

(a)$|\vec{a}|$ (b)$|\vec{a}|^2$ (c)1 (d)0

10 If $\vec{a} = \hat{i} + 2\hat{j} - 3\hat{k}$ and $\vec{b} = 2\hat{i} - 2\hat{j} - \hat{k}$ then $\vec{a}.\vec{b}$ is equal to

(a)1 (b)0 (c)-1 (d)3

2 Marks Questions

1. Adjacent sides of a parallelogram are given by $\hat{\imath} + 2\hat{\jmath} - \hat{k}$ and $3\hat{\imath} - \hat{\jmath} + 5\hat{k}$. Find a unit vector along a diagonal of the parallelogram.

2. Adjacent sides of a parallelogram are given by $6\hat{\imath} - \hat{\jmath} + 5\hat{k}$ and $\hat{\imath} + 5\hat{\jmath} - 2\hat{k}$. Find the area of parallelogram.

3. Find the area of triangle whose sides are given by the vectors $\hat{\imath} - 2\hat{\jmath} + \hat{k}$ and $4\hat{\imath} + \hat{\jmath} - 7\hat{k}$.

4. Find the value of p if the vectors $p\hat{\imath} + \hat{\jmath} + 4\hat{k}$ and $2\hat{\imath} - \hat{\jmath} + 3\hat{k}$ are perpendicular to each other.

5. Find a vector of magnitude 8units along $\vec{a} = 2\hat{\imath} - 4\hat{\jmath} + \hat{k}$

6. Find a unit vector along $\vec{a} = 5\hat{\imath} + 3\hat{\jmath} - 4\hat{k}$

7. If $\vec{a} = 2\hat{\imath} - 4\hat{\jmath} + \hat{k}$, $\vec{b} = 3\hat{\imath} - \hat{\jmath} - 5\hat{k}$ then find $|\vec{a} \times \vec{b}|$.

8. Find the projection of $\vec{a} = 2\hat{\imath} - 4\hat{\jmath} + \hat{k}$ on $\vec{b} = 3\hat{\imath} - \hat{\jmath} - 5\hat{k}$.

9. Find the area of parallelogram whose diagonals are given by vectors:

(i) $\vec{a} = 2\hat{i} + \hat{j} + \hat{k}$ & $\vec{b} = \hat{i} - \hat{k}$

(ii) $\vec{a} = \hat{i} + \hat{j} - 4\hat{k}$ & $\vec{b} = \hat{i} + 8\hat{j} + 2\hat{k}$

10. **Find the angle between the following vectors:**

(i) $-2\hat{i} - 2\hat{j} + 4\hat{k}$ and $-2\hat{i} + 4\hat{j} - 2\hat{k}$

(ii) $\hat{i} + 2\hat{j} + \hat{k}$ and $3\hat{i} + 2\hat{j} - 7\hat{k}$

3/4 Marks Questions

1. **For any two vectors $\vec{a}$ and $\vec{b}$ prove that $|\vec{a}\,.\vec{b}| \leq |\vec{a}||\vec{b}|$. Also write the name of inequality.**
2. **For any two vectors $\vec{a}$ and $\vec{b}$ prove that $|\vec{a} + \vec{b}| \leq |\vec{a}| + |\vec{b}|$. Also write the name of inequality.**
3. **Find the area of triangle whose vertices are :**

   (i) $A(2,3,5), B(3,5,8), C(2,7,8)$

   (ii) $A(1,2,4), B(3,1,-2), C(4,3,1)$

   (iii) $P(1,1,1), Q(1,2,3), R(2,3,1)$
4. **Let $\vec{a}$, $\vec{b}$, $\vec{c}$ be three vectors such that $|\vec{a}| = 3$, $|\vec{b}| = 4$, $|\vec{c}| = 5$ and each one of them is perpendicular to the sum of the other two, find $|\vec{a} + \vec{b} + \vec{c}|$.**
5. **If $\vec{a} = 5\hat{\imath} + \hat{\jmath} - 2\hat{k}$, $\vec{b} = 7\hat{\imath} + 2\hat{\jmath} - 3\hat{k}$, $\vec{c} = 2\hat{\imath} - 9\hat{\jmath} - \hat{k}$ find a vector of magnitude 7 units parallel to the vector $2\vec{a} - \vec{b} + 3\vec{c}$.**
6. **If $\vec{a} = \hat{\imath} + 4\hat{\jmath} + 2\hat{k}$, $\vec{b} = 3\hat{\imath} - 2\hat{\jmath} + 7\hat{k}$, $\vec{c} = 2\hat{\imath} - \hat{\jmath} + 4\hat{k}$, then find a vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$ and $\vec{c}.\vec{d} = 15$.**

Three Dimensional Geometry

Multiple Choice Questions(1 Marks)

1 **Direction ratios of straight line $\vec{r} = \hat{\imath} - 4\hat{\jmath} + 5\hat{k} + s(2\hat{\imath} - 3\hat{\jmath} + 2\hat{k})$ are :**

(a) $< 2, 3, 2 >$ **(b)** $< 2, -3, -2 >$ **(c)** $< -2, -3, 2 >$ **(d)** $< 2, -3, 2 >$

2 **Direction ratios of line given by $\frac{x-1}{3} = \frac{2y+6}{12} = \frac{1-z}{-7}$ are :**

(a) $< 3, 12, -7 >$ **(b)** $< 3, -6, 7 >$ **(c)** $< 3, 6, 7 >$ **(d)** $< 3, 6, -7 >$

3 **Direction ratios of a line passing through the points $(-2, 1, 0)$ & $(3, 2, 1)$ are**

(a) $< 5, 1, 1 >$ **(b)** $< -5, 1, -1 >$ **(c)** $< 5, -1, 1 >$ **(d)** $< -5, -1, 1 >$

4 **Which of the following sets of points are collinear :**

(a) $(1, 3, -4), (1, -2, 7)$ & $(3, 8, -11)$ **(b)** $(2, 3, -4), (2, -2, 3)$ & $(3, 5, -11)$

(c) $(2, 3, -4), (1, -2, 3)$ & $(3, 8, -11)$ **(d)** $(2, 3, -4), (1, -2, 3)$ & $(2, 8, 11)$

5 **Vector equation of the line $\frac{x+4}{5} = \frac{y-5}{3} = \frac{z-8}{-3}$ is**

(a) $\vec{r} = 4\hat{\imath} - 5\hat{\jmath} - 8\hat{k} + \mu(5\hat{\imath} + 3\hat{\jmath} - 3\hat{k})$ **(b)** $\vec{r} = -4\hat{\imath} + 5\hat{\jmath} + 8\hat{k} + \mu(5\hat{\imath} + 3\hat{\jmath} - 3\hat{k})$

(c) $\vec{r} = 5\hat{\imath} + 3\hat{\jmath} - 3\hat{k} + \mu(4\hat{\imath} - 5\hat{\jmath} - 8\hat{k})$ **(d)** $\vec{r} = 5\hat{\imath} + 3\hat{\jmath} - 3\hat{k} + \mu(-4\hat{\imath} + 5\hat{\jmath} - 8\hat{k})$

6 **Cartesian equation of the line $\vec{r} = 7\hat{\imath} - 5\hat{\jmath} + 3\hat{k} + \mu(9\hat{\imath} - \hat{\jmath} + 6\hat{k})$ is**

(a) $\frac{x+9}{7} = \frac{y-1}{-5} = \frac{z+6}{3}$ **(b)** $\frac{x-9}{7} = \frac{y+1}{-5} = \frac{z+6}{3}$ **(c)** $\frac{x+7}{9} = \frac{y-5}{-1} = \frac{z+3}{6}$ **(d)** $\frac{x-7}{9} = \frac{y+5}{-1} = \frac{z-3}{6}$

7 **Angle between the lines $\frac{x+1}{2} = \frac{y-5}{-1} = \frac{z}{1}$ and $\frac{x}{3} = \frac{y+7}{5} = \frac{z-8}{-1}$ is**

(a) $\pi/3$ **(b)** $\pi/2$ **(c)** $\pi/6$ **(d)** 0

8 **Direction cosines of a line making equal angles with coordinate axes are**

(a) $< \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} >$ **(b)** $< \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} >$ **(c)** $< 1, 1, 1 >$ **(d)** $< 0, 1, 0 >$

9 **Direction ratios of a line making equal angles with coordinate axes are**

(a) $< \frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} >$ **(b)** $< \frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}} >$ **(c)** $< 1, 1, 1 >$ **(d)** $< 0, 1, 0 >$

10 **If direction ratios of a line are $< 9, -6, 2 >$ then its direction cosines are**

(a) $< \frac{-9}{11}, \frac{6}{11}, \frac{2}{11} >$ **(b)** $< \frac{9}{11}, \frac{-6}{11}, \frac{2}{11} >$ **(c)** $< \frac{9}{7}, \frac{-6}{7}, \frac{2}{7} >$ **(d)** $< \frac{9}{5}, \frac{-6}{5}, \frac{2}{5} >$

True/False(1 Mark)

1) $3x + 2y - 9z = 19$ **is an equation of a line .**
2) **Direction cosine of y –axis are $< 0, 1, 0 >$.**
3) **Point $(2, -3, -5)$ lies on the line $\frac{x-2}{3} = \frac{y+3}{4} = \frac{z+5}{7}$.**
4) **The equation of a line parallel to the vector $2\hat{\imath} - \hat{\jmath} + 3\hat{k}$ and passing through the point $(5, -1, 3)$ is $\vec{r} = 5\hat{\imath} - \hat{\jmath} + 3\hat{k} + \mu(2\hat{\imath} - \hat{\jmath} + 3\hat{k})$.**
5) **The angle between two lines with direction ratios $< l, m, n >$ and $< a, b, c >$ is given by $\sin\theta = \frac{la+mb+nc}{\sqrt{a^2+b^2+c^2}\sqrt{l^2+m^2+n^2}}$.**
6) **Direction ratios of a line which passes through the points $(1, 2, 3)$ and $(4, 1, -2)$ are $< 3, -1, -5 >$.**
7) **Direction ratios of a line which makes equal angles with coordinate axes are $< 0, 0, 0 >$.**
8) **Direction ratios of line are $< 1, 0, 0 >$, then its direction cosines are $< 1, 0, 0 >$.**
9) **Direction ratios and direction cosines of a given line cannot be same numerically.**
10) **We can find the equation of a line if we have two points on the line.**

2/3 Marks Questions

1. **Find the equation of a line which passes through the points $(3, 6, -7)$ and $(5, -1, 4)$.**
2. **Find the direction cosines of a line passing through the points $(7, -1, 2)$ and $(3, 4, -7)$.**
3. **Find the direction ratios and direction cosines of a line which makes equal angles with the coordinate axes.**
4. **Find the direction cosines of sides of a triangle whose vertices are $(1, 2, -3), (9, -3, 7)$ and $(5, 3, -2)$.**
5. **Find the angle between the lines :**

(i) $\vec{r} = 3\hat{i} + 8\hat{j} + 3\hat{k} + \mu\left(3\hat{i} - \hat{j} + \hat{k}\right)$ & $\vec{r} = -3\hat{i} - 7\hat{j} + 6\hat{k} + \lambda\left(-3\hat{i} + 2\hat{j} + 4\hat{k}\right)$

(ii) $\vec{r} = 2\hat{i} - \hat{j} - \hat{k} + \mu\left(3\hat{i} - 5\hat{j} + 2\hat{k}\right)$ & $\vec{r} = \hat{i} + 2\hat{j} + \hat{k} + \lambda\left(\hat{i} - \hat{j} + \hat{k}\right)$

(iii) $\frac{x-1}{1} = \frac{y-2}{-1} = \frac{z-1}{1}$ & $\frac{x-2}{2} = \frac{y+1}{1} = \frac{z+1}{2}$

(iv) $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ & $\frac{x-2}{3} = \frac{y-4}{5} = \frac{z-5}{5}$

6. **Find the value of m if the lines $\frac{x+2}{3} = \frac{y-1}{2m} = \frac{z-2}{7}$ and $\frac{x-3}{4} = \frac{y-2}{7} = \frac{z+5}{8m}$ are perpendicular to each other.**

6/4 Marks Questions

1. **Find the shortest distance between the following pairs of lines :**

(i) $\vec{r} = 3\hat{i} + 8\hat{j} + 3\hat{k} + \mu\left(3\hat{i} - \hat{j} + \hat{k}\right)$ & $\vec{r} = -3\hat{i} - 7\hat{j} + 6\hat{k} + \lambda\left(-3\hat{i} + 2\hat{j} + 4\hat{k}\right)$

(ii) $\vec{r} = 2\hat{i} - \hat{j} - \hat{k} + \mu\left(3\hat{i} - 5\hat{j} + 2\hat{k}\right)$ & $\vec{r} = \hat{i} + 2\hat{j} + \hat{k} + \lambda\left(\hat{i} - \hat{j} + \hat{k}\right)$

(iii) $\frac{x-1}{1} = \frac{y-2}{-1} = \frac{z-1}{1}$ & $\frac{x-2}{2} = \frac{y+1}{1} = \frac{z+1}{2}$

(iv) $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ & $\frac{x-2}{3} = \frac{y-4}{5} = \frac{z-5}{5}$

(v) $\frac{3-x}{2} = \frac{8-2y}{-10} = \frac{z-1}{1}$ & $\frac{3x-6}{9} = \frac{5-y}{1} = \frac{4-2z}{8}$

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LINEAR PROGRAMMING

Multiple Choice Questions(1 Mark)

1 All points of feasible region are :
(a)infeasible solutions (b)feasible solutions (c)optimal solutions (d)none of these

2 Corner points of the feasible region are
(a)optimal solutions (b)useless points (c)infeasible solutions (d)none of these

3 Common area for each constraint is called :
(a)infeasible region (b)feasible region (c)useless area (d)none of these

4 Maximum value of $Z = 4x + 3y$ subject to the constraints $x + y \leq 4$, $x, y \geq 0$ is
(a)16 (b)12 (c)10 (d)20

5 Minimum value of $Z = 4x + 3y$ subject to the constraints $x + y \leq 4$, $x, y \geq 0$ is
(a)16 (b)12 (c)10 (d)20

6 Maximum value of $Z = 2x + 3y - 1$ subject to the constraints $x + y \leq 5$, $x, y \geq 0$ is
(a)14 (b)12 (c)10 (d)9

7 Minimum value of $Z = 5x + 3y + 2$ subject to the constraints $x + y \leq 7$, $x, y \geq 0$ is
(a)37 (b)35 (c)21 (d)23

8 Constraints of LPP are :
(a)Always quadratic (b)Always linear
(c)May be linear or quadratic depending on the problem (d)May be cubic some times

9 Objective function of LPP is
(a)Always quadratic (b)Always linear
(c)May be linear or quadratic depending on the problem (d)May be cubic some times

10 Minimum value of $Z = 5x + 3y + 2$ subject to the constraints $x + y \leq 7$, $x, y \geq 0$ on the point
(a)$(7, 0)$ (b)$(0, 7)$ (c)$(3, 4)$ (d)$(4, 3)$

True/False(1 Mark)

1) Point $(2, 3)$ is contained in the half plane $x + y \leq 0$.
2) Origin is contained in the half plane $2x + 3y \leq 12$.
3) Maximum value of $Z = 4x + y$ for the constraint $x + y \leq 12$, $x, y > 0$ is 48.
4) Any point outside the feasible region is called an infeasible solution.
5) When feasible region is bounded then objective function has both maximum and minimum values.
6) Subject to the constraints $x + y \leq 4, x \geq 0, y \geq 0$, maximum value of $Z = 3x + 4y$ is at $(0, 4)$.
7) The points within and on the boundary of feasible region do not represent the feasible solutions.
8) The problems which seeks to maximize or minimize profit or loss can be solved using linear programming.
9) The inequalities of a linear programming problem are not called constraints.
10) The maximum or minimum value of objective function is called optimal solution.

<u>4 Marks Questions</u>

Solve the following LPP graphically:

1. **Maximize & Minimize :**

    (i) $Z = 10x + 7y$ **subject to the constraints** $3x + y \leq 9,\ 3x + 2y \leq 12,\ x, y \geq 0.$

    (ii) $Z = x + 2y$ **subject to the constraints** $7x + 3y \leq 21,\ x + y \geq 3, x - y \leq 0,\ x, y \geq 0.$

    (iii) $Z = 4x + 2y$ **subject to the constraints** $8x + 9y \leq 72,\ 4x + y \geq 8,\ 2x - y \geq 0,\ x, y \geq 0.$

    (iv) $Z = 5x + 7y$ **subject to the constraints** $x + y \geq 4,\ x + 3y \leq 12,\ x - 2y \geq 0,\ x, y \geq 0.$

    (v) $Z = 3x + 6y$ **subject to the constraints** $x + y \leq 6, 2x + y \geq 6, 2x - y \leq 0, x, y \geq 0.$

    (vi) $Z = 4x + 5y$ **subject to the constraints** $x + y \leq 6, 2x + y \geq 6, x - y \geq 0, x, y \geq 0.$

    (vii) $Z = 8x + y$ **subject to the constraints** $x + y \leq 8, 2x + y \geq 8, x - 2y \leq 0, x, y \geq 0.$

    (viii) $Z = 3x + 7y$ **subject to the constraints** $x + y \leq 7, x + y \geq 3, x \leq 6, y \leq 6, x, y \geq 0.$

    (ix) $Z = 8x + 5y - 2$ **subject to the constraints** $x + y \leq 8, x + y \geq 5, x \leq 7, y \leq 7, x, y \geq 0$

    (x) $Z = 7x + 5y - 1$ **subject to the constraints** $x + y \leq 10, x + y \geq 5, x \leq 9, y \leq 9, x, y \geq 0$

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PROBABILITY

Multiple Choice Questions(1 Mark)

1 If $P(A) = \frac{1}{2}$, $P(B) = \frac{3}{8}$ and $P(A \cap B) = \frac{1}{5}$ then $P(A|B)$ is equal to :

(a)$\frac{2}{5}$ (b) $\frac{8}{15}$ (c) $\frac{2}{3}$ (d) $\frac{5}{8}$

2 If A and B are independent events and $P(A) = \frac{1}{2}$, $P(B) = \frac{3}{8}$ then $P(A \cap B)$ is equal to :

(a)$\frac{3}{4}$ (b) $\frac{3}{8}$ (c) $\frac{3}{16}$ (d) $\frac{1}{16}$

3 If $P(A) = \frac{1}{2}$, $P(B) = \frac{3}{8}$ and $P(A \cap B) = \frac{1}{5}$ then $P(B|A)$ is equal to :

(a)$\frac{2}{5}$ (b) $\frac{8}{15}$ (c) $\frac{2}{3}$ (d) $\frac{5}{8}$

4 Probability of getting even prime number on both dice ,on a throw of a pair of die, is :

(a)1/6 (b)2/35 (c)1/36 (d)5/36

5 If $P(A) = \frac{1}{2}$, $P(B) = \frac{3}{8}$ and $P(A \cup B) = \frac{4}{5}$ then $P(A|B)$ is equal to :

(a)$\frac{1}{5}$ (b) $\frac{8}{15}$ (c) $\frac{2}{3}$ (d) $\frac{5}{8}$

6 If E is any event then P(E) belongs to the interval :

(a)$(1, 10)$ (b) $(0, 1)$ (c) $[0, 1]$ (d) $[10, 20]$

7 If $P(E) = \frac{5}{7}$ then $P(not\ E)$ is

(a)$\frac{5}{7}$ (b)$\frac{7}{5}$ (c)$\frac{7}{2}$ (d)$\frac{2}{7}$

8 A coin is marked with head on both sides then on tossing the coin probability of getting head is :

(a)$\frac{1}{2}$ (b)0 (c)1 (d)2

9 Probability of getting an ace card on drawing one card from a well shuffled deck of 52 cards is :

(a)1/13 (b)1/4 (c)1/52 (d)none of these

10 Probability of 53 Mondays in a leap year is

(a)$\frac{2}{53}$ (b)$\frac{2}{7}$ (c)$\frac{1}{53}$ (d)$\frac{1}{7}$

11 Probability of 53 Mondays in a non-leap year is

(a)$\frac{2}{53}$ (b)$\frac{2}{7}$ (c)$\frac{1}{53}$ (d)$\frac{1}{7}$

12 If E is any event then $P(not\ E)$ belongs to the interval :

(a)$(1, 10)$ (b) $(0, 1)$ (c) $[0, 1]$ (d) $[10, 20]$

13 If three coins are tossed once, then getting at least one heads is

(a)$\frac{3}{8}$ (b)$\frac{7}{8}$ (c)$\frac{5}{8}$ (d)$\frac{1}{2}$

14 There are 3 red balls, 4 white balls and 7 blue balls in a bag. One ball is drawn at random from the bag. Probability of drawing a white ball is

(a)$\frac{2}{7}$ (b)$\frac{3}{14}$ (c)$\frac{7}{14}$ (d)0

15 There are 3 red balls, 4 white balls and 7 blue balls in a bag. One ball is drawn at random from the bag. Probability of drawing a green ball is

(a)$\frac{2}{7}$ (b)$\frac{3}{14}$ (c)$\frac{7}{14}$ (d)0

16 If E and F are independent events , then

(a)$P(E \cup F) = P(E) + P(F)$ (c)$P(E \cap F) = P(E) + P(F)$

(c) $P(E \cap F) = P(E).P(F)$ (d) $P(E \cap F) = 0$

Fill in the Blanks(1 Mark)

1) If $P(A) = \frac{1}{5}$ then $P(not\ A) =$ ______________
2) In a throw of a pair of dice probability of getting a doublet is ______________
3) Probability of occurrence of sure event $=$ ______________
4) Probability of occurrence of impossible event $=$ ______________
5) $P(A \cup B) = P(A) + P(B) -$ __________
6) $P(A) +$ ______ $= 1$
7) If A and B are independent events then $P(A \cap B) =$ ______________
8) If $P(A) = \frac{1}{2}$ and $P(B) = 0$ then $P(A|B)$ is _____________
9) If a dice is tossed once then probability of getting an odd prime number is ___________
10) Probability of any event is (numerically) always less than or equal to ___________

4 Marks Questions

1. If $P(A) = \frac{6}{11}, P(B) = \frac{5}{11}$ and $P(A \cup B) = \frac{7}{11}$ then find : $P(A \cap B), P(A/B) \&\ P(B/A)$
2. If $P(E) = 0.45,\ P(F) = 0.55\ \&\ P(E \cup F) = 0.75$ then find $P(E \cap F) \&\ P(E/F)$.
3. If $A \,\&\, B$ are independent events and :
   (i) If $P(A) = 0.4,\ P(A \cup B) = 0.7$ then find $P(B)$.
   (ii) if $P(A) = 0.5,\ P(A \cup B) = 0.7$ then find $P(B)$.
   (iii) If $P(A) = 0.35,\ P(A \cup B) = 0.60$ then find $P(B)$.
4. An urn contains 7 red and 4 blue balls. Two balls are drawn at random with replacement. Find the probability of getting : (a) 2 red balls, (b) 2 blue balls.
5. A bag contains 3 white and 5 black balls. Two balls are drawn at random without replacement. Determine the probability of getting the black balls.
6. A husband and wife appear in an interview for two vacancies in the same post. The probability of husband's selection is 1/7 and that of wife's is 1/5. Find the probability that (a) both get selected (b) only one of them get selected.
7. The probability of A hitting a target is 4/5 and that of B is 2/3. They both fire at the target. Find the probability that : (a) at least one of them will hit the target, (b) only one of them will hit the target.
8. A problem is given to 3 students whose chances of solving it are 1/3, 1/5 and 1/6. What is the probability that (i) exactly one of them may solve it, (ii) the problem will be solved.
9. A problem is given to 3 students whose chances of solving it are 1/3, 1/5 and 1/6. What is the probability that (i) exactly two of them may solve it, (ii) at least two of them will solve it , (iii) problem will be solved.
10. Two bags contain 6 red and 4 black balls, 3 red and 3 black balls. One ball is drawn at random from one of the bags and it is found to be red. Find the probability that it was drawn from first bag.
11. Two bags contain 7 red and 2 black balls, 3 red and 6 black balls. One ball is drawn at random from one of the bags and it is found to be red. Find the probability that it was drawn from first bag.
12. Two bags contain 5 red and 4 black balls, 7 red and 3 black balls. One ball is drawn at random from one of the bags and it is found to be red. Find the probability that it was drawn from second bag.
13. Two bags contain 6 red and 5 black balls, 7 red and 9 black balls. One ball is drawn at random from one of the bags and it is found to be red. Find the probability that it was drawn from second bag.

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Time Allowed : 3 Hours **MATHEMATICS-10+2** **Max. Marks :80**

Instructions:

1. All the questions are compulsory.
2. The question paper consists of 16 questions divided into 4 sections A,B,C and D.
3. Section A comprises of 3 questions :
   (i) Q.No.1 consists of 16 Multiple Choice Questions carrying 1 mark each.
   (ii) Q.No.2 consists of 8 Fill in the Blank type questions carrying 1 mark each.
   (iii) Q.No.3 consists of 8 True/False type questions carrying 1 mark each.
4. Section B comprises of 5 questions of 2 marks each.
5. Section C comprises of 5 questions of 4 marks each.
6. Section D comprises of 3 questions of 6 marks each.
7. Internal choice has been provided in three questions of 2 marks, three questions of 4 marks and three questions of 6 marks. You have to attempt only one of the alternatives in all such questions.
8. Use of calculator is not permitted.

Section – A

Q1 Choose the correct options in the following questions :

(i) Function $f: R \to R,\ f(x) = 3x - 5$ is : **1**
(a)one-one only (b)onto only (c)one-one and onto (d)none of these

(ii) Relation given by $R = \{(1,1),(2,2),(1,2),(2,1)\}$ is **1**
(a)reflexive only (b)symmetric only (c)transitive only (d) equivalence relation

(iii) $\cos^{-1}\left(\cos\frac{5\pi}{3}\right)$ is equal to : **1**
(a)$\frac{\pi}{5}$ (b) $\frac{2\pi}{3}$ (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{3}$

(iv) If $\begin{bmatrix} 1 & -x \\ 4 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 8 \\ 4 & -3 \end{bmatrix}$ then value of x is: **1**
(a)8 (b)-4 (c)3 (d)-8

(v) If order of matrix A is 2×3 and order of matrix B is 3×5 then order of matrix BA is : **1**
(a)5×2 (b) 2×5 (c) 5×3 (d) 3×2

(vi) If $f(x) = \begin{cases} kx + 1, & x \le 5 \\ 3x - 5, & x > 5 \end{cases}$ is continuous then value of k is : **1**
(a)$\frac{9}{5}$ (b) $\frac{5}{9}$ (c) $\frac{5}{3}$ (d) $\frac{3}{5}$

(vii) $\frac{d}{dx}\{\tan^{-1}(e^x)\}$is equal to : **1**
(a)$e^x \tan^{-1} e^x$ (b) $\frac{e^x}{1+e^{2x}}$ (c) 0 (d)$e^x \sec^{-1} x$

(viii) Critical point of the function $f(x) = x^2 - 10x + 2$ is : **1**
(a)$x = 4$ (b)$x = 6$ (c)$x = 5$ (d)$x = 2$

(ix) $\int 3x^2 dx$ is equal to : **1**
(a)$x + c$ (b)$x^2 + c$ (c)$x^3 + c$ (d)$x^4 + c$

(x) $\int_0^{\pi/2} \frac{\sin^{1/2} x}{\sin^{1/2} x + \cos^{1/2} x} dx$is equal to : **1**
(a)0 (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{4}$

(xi) Degree of differential equation $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 3y = 0$ is: **1**
(a)3 (b) 2 (c)1 (d)0

(xii) If $\vec{a}.\vec{b} = |\vec{a} \times \vec{b}|$ then angle between vector $\vec{a}$ and vector $\vec{b}$ is : **1**
(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{3}$

(xiii) If $\vec{a}.\vec{b} = 0$ then angle between vectors $\vec{a}$ and $\vec{b}$ is : **1**
(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{3}$

(xiv) Direction ratios of line given by $\frac{x-1}{3} = \frac{2y+6}{12} = \frac{1-z}{-7}$ are : **1**
(a) $< 3,12,-7 >$ (b) $< 3,-6,7 >$ (c) $< 3,6,7 >$ (d)$< 3,6,-7 >$

(xv) Common area for each constraint is called : **1**
(a)infeasible region (b)feasible region (c)useless region (d)main region

(xvi) If $P(A)=\frac{1}{2}$, $P(B)=\frac{3}{8}$ and $P(A\cap B)=\frac{1}{5}$ then $P(A/B)$ is equal to : 1

(a) $\frac{2}{5}$ (b) $\frac{8}{15}$ (c) $\frac{2}{3}$ (d) $\frac{5}{8}$

Q2 Fill in the blanks from the given options

(i) Value of $\sin^{-1}(1)$ is ___________ 1

(ii) If $A=[a_{ij}]_{2\times 3}$ such that $a_{ij}=i+j$ then $a_{11}=$__________ 1

(iii) If $\begin{vmatrix} x & 0 \\ 7 & 1 \end{vmatrix}=\begin{vmatrix} 3 & 0 \\ 7 & 2 \end{vmatrix}$ then $x=$ _______ 1

(iv) If $y=\cos x$ then at $x=0$, $\frac{dy}{dx}=$ ______________ 1

(v) $\int_0^5 dx=$__________ 1

(vi) Order of the differential equation $\frac{d^2y}{dx^2}-\left(\frac{dy}{dx}\right)^3+y=0$ is _________ 1

(vii) Direction ratios of x −axis are _______________ 1

(viii) Probability of occurrence of impossible event = ______________ 1

Q3 State true or false for the following statements :

(i) If A is a square matrix then $(A+A')$ is a skew-symmetric matrix. 1

(ii) If $y=10x$ then $\frac{dy}{dx}=0$. 1

(iii) If $y=\tan x$ then $\frac{dy}{dx}=\sec^2 x$ 1

(iv) $\int dx=x^2+c$ 1

(v) $xdy-ydx=0$ is a variable separable type of differential equation. 1

(vi) Scalar product of two perpendicular vectors is zero. 1

(vii) The point $(3,-4,2)$ lies on the $y-$ axis. 1

(viii) If $P(E)=0.4$ then $P(not\ E)=0.6$ 1

Section – B

Q4 Given a matrix $A=\begin{bmatrix} 3 & 5 \\ 2 & 7 \end{bmatrix}$,show that matrix $P=A+A'$ is a symmetric matrix. 2

Q5 Find the interval in which function $f(x)=x^2+2x-7$ is increasing. 2

OR

If $y=\log x$, then find $\frac{d^2y}{dx^2}$ 2

Q6 Evaluate $\int_{\pi/6}^{\pi/3}\frac{dx}{1+\sqrt{\tan x}}$. 2

OR

Evaluate $\int\sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}\,dx$ 2

Q7 Using integration find the area bounded by the parabola $y^2=4x$ straight lines $x=1, x=4$ in the first quadrant. 2

Q8 Find the value of m if the vectors $\vec{a}=2\hat{\imath}-\hat{\jmath}-m\hat{k}$ and $\vec{b}=5\hat{\imath}+2\hat{\jmath}-\hat{k}$ are perpendicular to each other. 2

OR

Find the angle between the lines : $\frac{x-2}{1}=\frac{y-3}{2}=\frac{z-7}{-5}$ and $\frac{x+5}{3}=\frac{y-2}{2}=\frac{z-6}{4}$ 2

Section – C

Q9 Show that function : $\mathbb{R}\to\mathbb{R}$, $f(x)=\frac{4-3x}{2}$ is one-one and onto. 4

Q10 If $y=x^{\sin x}+(\sin x)^x$ then find $\frac{dy}{dx}$. 4

OR

Find the general solution of the differential equation $(\tan^{-1}y-x)dy=(1+y^2)dx$. 4

Q11 Evaluate $\int_0^{\pi/2} \log \sin x \, dx$. **4**

OR

Evaluate $\int \left[\log(\log x) + \frac{1}{(\log x)^2}\right] \; dx$ **4**

Q12 Solve the following linear programming problem graphically:
Maximize and minimize $Z = 4x + 3y$ subject to the constraints **4**

$$x + y \leq 8, \qquad 4x + y \geq 8, \qquad x - y \geq 0, \qquad x \geq 0 \qquad , y \geq 0$$

Q13 Probability of solving a specific problem independently by A and B are 1/2 and 1/3 respectively. If both try to solve the problem independently, find the probability that : **4**
(i)the problem is solved (ii)exactly one of them solves the problem

OR

In an examination , 20 questions of true-false type are asked. Suppose a student tosses a fair coin to determine his answer to each question. If coin falls heads, he answers true and if it falls tails, he answers false. Find the probability that he answers at least 12 questions correctly. **4**

Section – D

Q14 (a) Express the matrix $A = \begin{bmatrix} 2 & 2 & 5 \\ 3 & 9 & 5 \\ 8 & 7 & 1 \end{bmatrix}$ as a sum of a symmetric matrix and a skew-symmetric matrix. **4**

(b) If $A = \begin{bmatrix} 5 & -2 \\ 4 & 8 \end{bmatrix}$ and $B = \begin{bmatrix} -4 & 0 \\ 3 & 2 \end{bmatrix}$ then show that $(AB)' = B'A'$ **2**

OR

Solve the following system of linear equations by matrix method : **6**

$$2x - 4y + 5z = 3 \qquad 3x + y - 4z = 0 \qquad x + y - z = 1$$

Q15 Show that height of the cylinder of maximum volume that can be inscribed in a sphere of 30 cm is $\frac{60}{\sqrt{3}}$ cm. **6**

OR

Solve $\int \frac{1}{x^4+1} dx$ **6**

Q16(a) Find the projection of the vector $\vec{a} = 3\hat{\imath} - 2\hat{\jmath} + 7\hat{\jmath}$ on the vector $\vec{b} = 6\hat{\imath} + \hat{\jmath} - 2\hat{k}$ **2**

(b) Find any daigonal of the parallelogram whose adjacent sides are given by the vectors $\vec{a} = 5\hat{\imath} + 2\hat{\jmath} + \hat{k}$ and $\vec{b} = \hat{\imath} + 9\hat{\jmath} + 2\hat{k}$ .Also find the area of the parallelogram. **4**

OR **6**

A line makes angles α, β, γ and δ with the diagonals of a cube, prove that **6**

$$\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma + \cos^2 \delta = \frac{4}{3}$$