QUESTION BANK

Class : 10+1 & 10+2

(Mathematics)

Question Bank for +1 and +2 students for the subject of Mathematics is hereby given for the practice. While preparing the questionnaire, emphasis is given on the concepts, students, from the examination point of view.

We hope that you might appreciate this question bank. We welcome suggestions to improve the question bank.

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CLASS – 10+1
MATHEMATICS

CLASS – 10+1

CONTENTS

Sets (Marks -1)

Q.1. $(A \cup B)^C$ is equation to

(a) $A^C \cup B^C$ (b) $A^C \cap B^C$

(c) $A^C - B^C$ (d) None of these

Q.2. The set of girls in a boy's school is

(a) a null set (b) singleton set

(c) a finite set (d) not a well defined collection

Q.3. The set of principals in a school is

(a) a null set (b) a singleton set

(c) an infinite (d) None of these

Q.4. Solution set of equation $x^2 - 5x + 6 = 0$ in roster form is

(a) {-2, -3} (b) {2, 3}

(c) {-3, 2} (d) {-2, 3}

Q.5. Set of even prime numbers is

(a) a Null set (b) a Singleton set

(c) a finite set (d) an infinite set

Q.6. The se $A = \{u : u \in R, u^2 = 49, 2u = 14\}$ is

(a) ϕ (b) {7}

(c) {-7} (d) {-7,7}

Q.7. In a college of 300 students, every student reads 5 newspapers and every newspaper is read by 60 students. The number of newspapers is

(a) At least 30 (b) at most 20

(c) exactly 25 (d) None of the above

Q.8. If S = {0,1,5,4,7} then the total number of subsets of S is

(a) 64 (b) 32

(c) 40 (d) 20

Q.9. if A $\leq$ B then B $\cup$ A equals

(a) A (b) B $\cap$ A

(c) B (d) None of these

Q.10. Set of odd natural numbers divisible by 2 is

(a) null set (b) a singleton set

(c) a finite set (d) an infinite set

Q.11. The set of $A = \{x : x \in R, x^2 = 16 \text{ and } 2x = 6\}$ is equals

(a) ϕ (b) {14,3,4}

(c) {3} (d) {4}

Q.12. If A & B are any two sets then $A \cap (A \cup B)$ equals

(a) A (b) B

(c) A^C (d) B^C

CHAPTER – SETS

(4 Marks Questions)

1. Let $A = \{\{1,2,3\},\{4,5\},\{6,7,8\}\}$

Determine which of the following is true or false :

(*a*) $1 \in A$ (*b*) $\{1, 2, 3\} \subset A$ (*c*) $\{6, 7, 8\} \in A$ (*d*) $\{\{4, 5\}\} \subset A$

(*e*) $\phi \in A$ (f) $\phi \subset A$ (g) $\{6, 7, 8\} \subset A$ (h) $5 \in A$

2. If $A = \{2, 3\}$, $B = \{x : x$ is a root of $x^2 + 5x + 6 = 0\}$, then find

(i) $A \cup B$

(ii) $A \cap B$

(iii) Are they equal sets?

(iv) Are they equivalent sets?

3. Let U = {1, 2, 3, 4, 5, 6, 7, 8,9}, A = {1, 2, 3, 4}, B = {2, 4, 6, 8}, Find :

(*a*) A^C (*b*) B^C (*c*) $(A^C)^C$ (*d*) $(A \cup B)^C$

4. If A = {1, 2,}, B = {4, 5, 6} and C = {7, 8, 9}, verify that :

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

5. Out of 20 members in a family, 11 like to take tea and 14 like coffee. Assume that each one likes at least one of the two drinks. How many like :

(*a*) both tea and coffee (*b*) only tea and not coffee (*d*) only coffee and not tea

6.(i) Write the following sets in set builder form :

A = {1, 3, 5, 7, 9}, E = {1, 5, 10, 15……}

(ii) Write the following sets in Roster Form :

A = {x : x is an integer and $-3 < x < 7$},

B = {x : x is a natural number less than 6}

7. Let A={1,2}, B= {1,2,3,4}, C={5,6} and D={5,6,7,8}. Verify that

(a) $A\times(B\cap C)=(A\times B)\cap(A\times C)$

(b) $(A\times C)\subset(B\times D)$

8. Out of 20 members in a family, 11 like to take tea and 14 like coffee. Assume that each one likes at least one of the two drinks. How many like

(i) Both tea and coffee.

(ii) Only tea and not coffee.

9. Prove that

(i) $A\subset B \Leftrightarrow B^C\subset A^C$

(ii) $B\subset A \Rightarrow A\cup B=A$

10. Prove that

(i) $A^C-B^C = B-A$

(ii) $B-A = B\cap A^C$

CHAPTER-RELATIONS AND FUNCTIONS

(1 mark question)

Q.1 Let $R = \{(1,3),(4,2),(2,4),(2,3),(3,1)\}$ be a relation on the set $A = \{1,2,3,4\}$. The relation R is

(A) a function (B) transitive

(C) not symmetric (D) reflexive.

Q.2 Let $R = \{(3,3),(6,6),(9,9),(12,12),(6,12),(3,9),(3,12),(3,6)\}$ be a relation on the set $A = \{3,6,9,12\}$. The relation is

(A) reflexive only (B) reflexive and transitive only

(C) reflexive and symmetric only (D) an equivalence relation

Q.3 Let R be the real number. Consider the following subsets of RxR

S = $\{(x,y): y = x+1 \text{ and } 0 < x < 2\}$

T = $\{(x,y): x - y \text{ is an integer }\}$

which one of the following is true?

(A) T is an equivalence relation on R but S is not.

(B) Neither S nor T is an equivalence relation on R.

(C) Both S and T are equivalence relations on R

(D) S is an equivalence relation on R but T is not.

Q.4 Let f(x) = [x] then f $\left(\frac{-3}{2}\right)$ is equal to :

(*a*) –3 (*b*) –2 (*c*) –1.5 (*d*) None of these

Q.5 Range of f(x) = $x^2 + 2$, where x is a real number, is :

(a) $[2, \infty)$ (b) $(2, \infty]$ (c) $(2, \infty)$ (d) $[2, \infty]$

Q.6 The domain of $f(x)=\sqrt{1+\log_e(1-x)}$ is :

(a) $-\infty < x \le 0$ (b) $-\infty \le x \le \frac{e-1}{e}$ (c) $-\infty < x \le 1$ (d) $x \ge 1-e$

Q.7 For real x, let $f(x)=x^3+5x+1$ then :

(a) f is onto R but not one-one (b) f is one-one and onto R

(c) f is neither one-one nor onto R (d) f is one-one but not onto R

Q.8 If $f(x)=x^2-\frac{1}{x^2}$, then find the value of $f(x)+f\left(\frac{1}{x}\right)$

(a) 1 (b) 0 (c) $\frac{1}{2}x^2$ (d) $\frac{1}{2x^2}$

Q.9 Find the domain of the function

$$f(x)=\sqrt{x^2-7x+10}$$

a) $(2,5)$ b) $[2,\infty)$ c) $(-\infty,2] \cup [5,\infty)$ d) $(-\infty,2)\cup[5,\infty)$

Q.10. Let $f : R \to R$ be defined as $f(x)=3x$. Then

(a) f is one-one onto,

(b) f is many one-onto

(c) f is one – one but not onto

(d) f is neither one-one nor onto

(4 marks question)

Q.1 If $G = \{7,8\}$ and $H = \{5,4,2\}$ find G x H and H x G.

Q.2 If $P\{1,2\}$ form the set $P\times P\times P$.

Q.3 Let $A = \{1,2,3,4\}$ and $B = \{5,7,9\}$

Determine :

(i) $A\times B$ and represent it graphically.

(ii) $B\times A$ and represent it graphically.

(iii) Is $n(A\times B) = n(B\times A)$

Q.4 Let $A=\{1,2,3\}$, $B=\{2,3,4\}$ and $C=\{4,5\}$ verify that

$A\times(B\cap C) = (A\times B)\cap(A\times C)$

Q.5 If $A=\{4,9,16,25\}$, $B=\{1,2,3,4\}$ and R is the relation "is square of" from A to B. Write down the set corresponding to R. Also find the domain and range of R.

Q.6 If R is a relation "is divisor of" from the set $A=\{1,2,3\}$ to $B=\{4,10,15\}$, write down the set of ordered pairs corresponding to R.

Q.7 Let $A=\{1,2\}$ and $B=\{3,4\}$. Find the number of relations from A to B.

Q.8 Let $N\rightarrow N$ be defines by $f(x)=3x$. Show that f is not an onto function.

Q.9 If 'f is a real function defined by $f(x)=\dfrac{x-1}{x+1}$ then prove that $f(2x)=\dfrac{3f(x)+1}{f(x)+3}$

Q.10 If $f(x)=\dfrac{1}{2x+1}, x\neq\dfrac{-1}{2}$, then show that $f(f(x))=\dfrac{2x+1}{2x+3}, x\neq\dfrac{-3}{2}$

Q.11 The function 't' which maps temperature in Celsius into temperature in Fahrenheit is defined by $t(c)=\frac{9c}{5}+32$ find (i) t(0) (ii) t(28) (iii) t(-10) (iv) the value of c when t(c)=212

Q.12 Find the domain of the function

$$f(x)=\frac{x^2+3x+5}{x^2-5x+4}$$

Q.13 The function f is defined by:

$$f(x)=\begin{vmatrix} 1-x, & x<0 \\ 1, & x=0 \\ x+1, & x>0 \end{vmatrix}$$

Draw the graph of $f(x)$

Q.14 Let A={9,10,11,12,13} and $f : A \rightarrow N$ be defined by

$f(x)$ = The highest prime factor of n. Find the range of f.

CHAPTER – TRIGONOMETRIC FUNCTIONS

(1 Mark Questions)

1. Radian measure of $40^0 20'$ is :

(a) $\frac{121}{540}$ radians (b) $\frac{540}{121}$ radians (c) $\frac{121}{540}\pi$ radians (d) None of these

2. Radian measure of 25^0 is :

(a) 25π (b) $\frac{26}{9}$ (c) $\frac{26}{9\pi}$ (d) None of these

3. Value of sin 765^0 is :

(a) 1 (b) $\sqrt{2}$ (c) $\frac{1}{\sqrt{2}}$ (d) 765^0

4. The principal solution of tan x = $\sqrt{3}$ is :

(a) $\frac{\pi}{3}$ (b) $\frac{4\pi}{3}$ (c) $\frac{2\pi}{3}$ (d) $\frac{5\pi}{3}$

5. The most general solution of tan θ = –1, cos θ = $\frac{1}{\sqrt{2}}$ is :

(*a*) $\theta = n\pi + \frac{7\pi}{4}$ (*b*) $\theta = n\pi + (-1)^n \frac{7\pi}{4}$

(*c*) $\theta = 2n\pi + \frac{7\pi}{4}$ (*d*) None of these

6. The value of $\cos 52^0 + \cos 68^0 + \cos 172^0$ is :

(*a*) 1 (*b*) 0 (*c*) –1 (*d*) 3

7. The equation $\sqrt{3}\sin x + \cos x = 4$ has :

(*a*) only one solution (*b*) two solutions (*c*) infinite many solutions (*d*) no solution

8. The value of $\cos 15^0 \cos 7\frac{1}{2}^0 \sin 7\frac{1}{2}^0$ is :

(*a*) $\frac{1}{2}$ (*b*) $\frac{1}{8}$ (*c*) $\frac{1}{4}$ (*d*) $\frac{1}{16}$

9. $Sin 47^0 + \sin 61^0 - \sin 11^0 - \sin 25^0 =$

(*a*) $\sin 7^0$ (*b*) $\cos 7^0$ (*c*) $\sin 36^0$ (*d*) $\cos 36^0$

10. The period of the function sin 3*x* is

(a) $\frac{\pi}{3}$ (b) $\frac{2\pi}{3}$ (c) 3π (d) None of these

(2 Marks Questions)

1. If in two circles, arcs of the same length subted angles of 60^0 and 75^0 at the centre, find the ratio of their radii.

2. Find the angle between the minute hand and the hour hand of a clock when the time is 5:20.

3. Prove that : (*a*) $\sec^4 A - \sec^2 A = \tan^4 A + \tan^2 A$ (*b*) $\tan^2\theta - \sin^2\theta = \tan^2\theta \sin^2\theta$

4. Prove that : $\frac{\text{cosec}\,\theta}{\text{cosec}\,\theta - 1} + \frac{\text{cosec}\,\theta}{\text{cosec}\,\theta + 1} = 2\sec^2\theta$

5. If $\cot\theta = \frac{-12}{5}$ and 'θ' lies in the second quadrant, find the values of other five functions.

6. Prove that : $\sin^2\frac{\pi}{6} + \cos^2\frac{\pi}{3} - \tan^2\frac{\pi}{4} = \frac{-1}{2}$

7. Prove that : $3\cos^2\frac{\pi}{4} + \sec\frac{2\pi}{3} + 5\tan^2\frac{\pi}{3} = \frac{29}{2}$

8. Prove that : $\tan 70^0 = \tan 20^0 + 2\tan 50^0$

9. Find the principal solutions of the following :

(a) $\tan x = \sqrt{3}$ (b) $\sec x = 2$

10. Prove that the equation $\cos\theta = x + \frac{1}{x}$ is impossible if x be real.

11. Prove that $\sin 150^o \cos 120^o + \cos 330^o \sin 660^o = -1$

12. Simplify the following $\sin(90^0+\theta)\tan(270^0+\theta)\cot(90^0+\theta)\mathrm{cosec}(270^0+\theta)$

13. If $\theta+\phi = 45^0$ prove that $(1+\tan\theta)(1+\tan\phi) = 2$

14. Prove that $\sin 70^0\cos 10^0 - \cos 70^0 \sin 10^0 = \frac{\sqrt{3}}{2}$

15. Prove that $\tan 70^0 = \tan 20^0 + 2\tan 50^0$

16. Prove that $3\cos^2\frac{\pi}{4} + \sec\frac{2\pi}{3} + 5\tan^2\frac{\pi}{3} = \frac{29}{2}$

17. Prove that $\sin^2\frac{\pi}{6} + \cos^2\frac{\pi}{3} - \tan^2\frac{\pi}{4} = \frac{-1}{2}$

18. $\sin^2 A\cos^2 B + \cos^2 A\sin^2 B + \cos^2 A\cos^2 B + \sin^2 A\sin^2 B = 1$

19. $\sqrt{\frac{1+\sin\theta}{1-\sin\theta}} = \sec\theta + \tan\theta$

20. $\frac{1-\cos\theta}{\sin\theta} = \frac{\sin\theta}{1+\cos\theta}$

21. Prove that $\sin^2 6x - \sin^2 4x = \sin 2x \sin 10x$

22. Write down the values of $\cos 68^0\cos 8^0 + \sin 68^0 \sin 8^0$

(4 Marks Question)

1. In any triangle ABC, prove that : $\sin\frac{B-C}{2}=\frac{b-c}{a}\cos\frac{A}{2}$

2. In any triangle ABC, prove that : $\frac{\cos A}{a}+\frac{\cos B}{b}+\frac{\cos C}{C}=\frac{a^2+b^2+c^2}{2abc}$

3. Prove that : $\tan 9^0-\tan 27^0-\tan 63^0+\tan 81^0=4$

4. Show that $\sqrt{2+\sqrt{2+\sqrt{2+2\cos 8\theta}}}=2\cos\theta$

5. Prove that $(\cos\alpha+\cos\beta)^2+(\sin\alpha+\sin\beta)^2=4\cos^2\frac{\alpha-\beta}{2}$

6. Prove that $\sin 10^0\sin 50^0\sin 60^0\sin 70^0=\frac{\sqrt{3}}{16}$

7. Prove that $\frac{\tan 5\theta+\tan 3\theta}{\tan 5\theta-\tan 3\theta}=4\cos 2\theta\cos 4\theta$

8. Prove that $\tan 4\theta=\frac{4\tan\theta(1-\tan^2\theta)}{1-6\tan^2\theta+\tan^4\theta}$

9. If $A+B+C=\pi$ prove that

   $\sin 2A+\sin 2B+\sin 2C=4\sin A\sin B\sin C$

10. Prove that $\sin\frac{\pi}{5}\sin\frac{2\pi}{5}\sin\frac{3\pi}{5}\sin\frac{4\pi}{5}=\frac{5}{16}$

11. Find the values of other trigonometric function

    (i) $\cot x=\frac{3}{4}$, x lies in third quadrant

    (ii) $\tan x=-\frac{5}{12}, x$ lies in second quadrant

CHAPTER – PRINCIPLE OF MATHEMATICAL INDUCTION

(4 Marks Questions)

1. By using the Principle of mathematical induction $3^{2n} -1$ is divisible by 8 for all $n \in N$
2. By Principle of Mathematical Induction, prove that :

$$1^2 + 2^2 + 3^2 + ... + n^2 = \frac{1}{6} n(n+1)(2n+1)$$

3. Prove that $10^{2n-1}+1$ is divisible by 11 for all $n \in N$
4. For every positive integer 'n', prove that $7^n - 3^n$ is divisible by 4.
5. By principle of mathematical Induction, prove that :

$$\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + ... + \frac{1}{n(n+1)} = \frac{n}{n+1} \text{ for all } n \geq 1.$$

6. By Principle of Mathematical induction, Prove that

$$1^2 + 2^2 .... n^2 = \frac{n(n+1)(2n+1)}{6}$$

7. By principle of Mathematical induction, prove that

$$\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} .... + \frac{1}{n(n+1)} = \frac{n}{n+1} \text{ for all n} \geq 1$$

8. By Principle of Mathematical induction. Prove that $1+3+3^2+...3^{n-1} = \left(\frac{3^n-1}{2}\right)$ for all $n \in N$

9. Prove the rule of exponents : $(ab)^n = a^n b^n$

   by using principle of mathematical Induction for every natural number.

CHAPTER – COMPLEX NUMBERS AND QUADRATIC EQUATIONS

(1 mark Question)

1. i^{-35} is :

(*a*) i (*b*) 1 (*c*) 0 (*d*) –i

2. Solution of $x^2 + 2 = 0$ is :

(*a*) –2 (*b*) 2 (*c*) $\pm\sqrt{2}$ (*d*) $\pm\sqrt{2}i$

3. Complex conjugate of 3i – 4 is :

(*a*) 3i + 4 (*b*) –3i – 4 (*c*) –3i + 4 (*d*) None of these

4. Additive inverse of complex number 4 – 7i is:

(*a*) 4 + 7i (*b*) –4 + 7i (*c*) –4 – 7i (*d*) None of these

5. The imaginary part of $\frac{-1}{5}+\frac{i}{5}$ is :

(*a*) zero (*b*) $\frac{-1}{5}$ (*c*) $\frac{1}{5}$ (*d*) None of these

6. The value of $i^{13}+i^{14}+i^{15}+i^{16}$ is :

(*a*) i (*b*) –i (*c*) zero (*d*) –1

7. $i^{57}+\frac{1}{i^{125}}$ equals :

(*a*) 0 (*b*) 2i (*c*) –2i (*d*) 2

8. The complex number z = x + iy, which satisfies the equation $\left|\frac{z+5i}{z-5i}\right|=1$, lies on :

(*a*) The line y = 5 (*b*) a circle through the origin

(*c*) the x – axis (*d*) None of these

9. The modulus of $\frac{1-i}{3+i}+\frac{4i}{5}$ is :

(*a*) $\sqrt{5}$ units (*b*) $\frac{\sqrt{11}}{5}$ units (*c*) $\frac{\sqrt{5}}{5}$ units (*d*) $\frac{\sqrt{12}}{5}$ units

10. The conjugate of a complex number is $\frac{1}{1-i}$. Then that complex number is :

(*a*) $\frac{1}{i-1}$ (*b*) $\frac{-1}{i-1}$ (*c*) $\frac{1}{i+1}$ (*d*) $\frac{-1}{i+1}$

(2 Mark Questions)

1. Solve the equation $\sqrt{5}x^2+x+\sqrt{5}=0$
2. Solve the equation $x^2-7ix-12=0$

3. Find the conjugate of $\frac{(3-2i)(2+3i)}{(1+2i)(2-i)}$

4. Prove that $1+l^{10}+l^{20}+i^{30}$ is real number

5. Express the complex no. i^9+i^{19} in the form of $a+ib$

6. Find the multiplicative inverse of 2-3i

7. Express $(2+7i)^3$ in the form a+ib.

8. Evaluate $\left[i^{18}+\left(\frac{1}{i}\right)^{25}\right]^3$

9. Find the modulus of $\frac{1+i}{1-i}-\frac{1-i}{i+i}$

10. If $\left(\frac{1+i}{1-i}\right)^m=1$ then fine the least +ve integral value of m.

(6 Mark Questions)

1. Show that a real value of x will satisfy the equation $\frac{1-ix}{1+ix}=a-ib$ if $a^2+b^2=1$, where a, b are real.

2. If $\frac{a+ib}{c+id}=x+iy$, show that $x^2+y^2=\frac{a^2+b^2}{c^2+d^2}$

3. If $(x+iy)^3=u+iv$, then show that : $\frac{u}{x}+\frac{v}{y}=4(x^2-y^2)$

4. Find the modulus and the argument of the complex number $z=-\sqrt{3}+i$

5. If z_1, z_2 are complex numbers, such that $\frac{2z_1}{3z_2}$ is purely imaginary number, find $\left|\frac{z_1-z_2}{z_1+z_2}\right|$

6. Convert into polar form : $\frac{1+7i}{(2-i)^2}$

7. Solve : $x^2-(3\sqrt{2}+2i)x+6\sqrt{2}i=0$

8. If $|z|=1$, prove that $\left|\frac{z-1}{z+1}\right|$ $(z\neq-1)$ is purely imaginary number. What will you conclude if $z = 1$.

9. Convert into polar form : $z = \dfrac{i-1}{\cos\dfrac{\pi}{3} + i\sin\dfrac{\pi}{3}}$

10. If $(x+iy) = \dfrac{a+ib}{c+id}$, then prove : $(x-iy) = \dfrac{a-ib}{c-id}$, and $x^2 + y^2 = \dfrac{a^2+b^2}{c^2+d^2}$

CHAPTER – LINEAR IN EQUATIONS

(6 Marks Questions)

1. Solve the following inequations and show the graph on number line :

(*a*) $3x-6<0$ (*b*) $-3x+9\le 0$ (*c*) $7x+5>33$ (*d*) $5x-15\ge 0$

2. Solve the following inequations and show that graph on number line :

(*a*) $\frac{x-3}{x-5}>0$ (*b*) $\frac{x}{4}<\frac{5x-2}{3}-\frac{7x-3}{5}$

3. Solve the following system of inequations :

$$\frac{5x}{4}+\frac{3x}{8}>\frac{39}{8} \text{ and } \frac{2x-1}{12}-\frac{x-11}{3}<\frac{3x+1}{4}$$

4. Solve the following system of inequations :

$$2(2x+3)-10<6(x-2) \text{ and } \frac{2x-3}{4}+6\ge 2+\frac{4x}{3}$$

5. Solve graphically :

(i) $|x|<2$ (ii) $|y|\ge 3$

6. Find the region enclosed by the following inequations

$$x+y-2\le 0,\ 2x+y-3\le 0,\ x\ge 0,\ y\ge 0$$

7. Find the region for following inequation :

$$x+y\ge 0,\ 2x+y\le 4,\ x\ge 0 \text{ and } y\ge 0$$

8. Solve the following system of inequalities graphically:

$4x+3y\le 60,\ y\ge 2x,\ x\ge 3,\ x, y\ge 0$

CHAPTER – PERMUTATION AND COMBINATION

Multiple Choice Questions : **(1 Mark Questions)**

1. 7! ÷ 5! is :

(*a*) 7! (*b*) 2! (*c*) 42 (*d*) 24

2. The value of is $\frac{12!}{10!2!}$:

(*a*) 42 (*b*) 66 (*c*) 76 (*d*) 45

3. The value of $^{15}C_{11} \div {}^{15}C_{10}$ is :

(*a*) $\frac{15}{11}$ (*b*) $\frac{15}{10}$ (*c*) $\frac{5}{11}$ (*d*) $\frac{5}{10}$

4. If $^4P_n = 5\ {}^4P_3$, then n is :

(*a*) 8 (*b*) 6 (*c*) 7 (*d*) 5

5. If n = 7 and r = 5, then the value of ^nC_r is :

(*a*) 21 (*b*) 42 (*c*) 35 (*d*) 75

6. If n = 8 and r = 3 then the value of ^nP_r is :

(*a*) 140 (*b*) 336 (*c*) 40 (*d*) 85

7. Evaluate : $^{10}C_1 + {}^{10}C_2 + {}^{10}C_3 + \dots\dots\dots {}^{10}C_{10}$

(*a*) 1000 (*b*) 1023 (*c*) 1050 (*d*) 1010

8. The number of ways in which 6 men and 5 women can sit at a round table if no two women are to sit together is given by :

(*a*) 30 (*b*) 5! × 4! (*c*) 7! × 5! (*d*) 6! × 5!

9. If $^{2n+1}P_{n-1} : {}^{2n-1}P_n = 3:5,$ then the value of n equal :

(*a*) 4 (*b*) 3 (*c*) 2 (*d*) 1 (*e*) 5

10. If $n_{c_2} - n_{c_1} = 35$ then the value of *n* equal :

(*a*) -10 (*b*) 10 (*c*) 7 (*d*) -7

(4 Marks Questions)

1. Find n such that $\frac{{}^{n}P_4}{{}^{n-1}P_4} = \frac{5}{3}$, $n > 4$.

2. In how many ways can 9 examination papers be arranged so that the best and the worst papers never come together?

3. The letters of the word 'RANDOM' are written in all possible orders and these words are written out as in dictionary. Find the rank of the word 'RANDOM'.

4. How many natural numbers less than 1000 can be formed with the digits 1, 2, 3, 4 and 5 if (a) no digit is repeated (b) repetition of digits is allowed.

5. Find out how many arrangements can be made with the letters of the word 'MATHEMATICS'. In how many ways can consonants occur together?

6. In how many ways can 5 persons – A, B, C, D and E sit around a circular table if :

(*a*) B and D sit next to each other. (*b*) A and D do not sit next to each other.

7. How many triangles can be obtained by joining 12 points, five of which are collinear?

8. If m parallel lines in a plane are intersected by a family of n parallel lines, find to number of parallelograms formed.

9. What is the number of ways of choosing, 4 cards from a pack of 52 playing cards? In how many of these :

(*a*) four cards are of the same suits (*b*) are face cards

10. Prove that

$$\frac{2^n(1.3.5....(2n-1))}{n!} = {}^{2n}C_n$$

CHAPTER – BIONOMIAL THEOREM

(2 Marks Questions)

1. Find the number of terms in the expansion of $(2x-3y+4z)^n$

2. Expand $\left(\frac{x}{3}+\frac{1}{x}\right)^5$ $x \neq 0$

3. Determine the two middle terms in the expansion of $(x^2+a^2)^5$

4. Find the term containing x^3, if any, in $\left(3x-\frac{1}{2x}\right)^8$

5. Find the term, which is independent of *x* in the expansion of $\left(x^2+\frac{1}{x}\right)^9$

6. For what value of *m*, the coefficients of $(2m+1)^{th}$ and $(4m+5)^{th}$ terms in the expansion of $(1+x)^{10}$ are equal.

7. Which term is independent of *x* in the expansion of $\left(2x^2+\frac{1}{x}\right)^{12}$.

8. Evaluate $\sum_{r=1}^{n} n_{c_r} 2^r$

9. What is the fourth term in the expansion of $\left(3-\frac{x^3}{6}\right)^7$?

10. Find the middle term in the expansion of $\left(3-\frac{x^5}{2}\right)^9$

11. Find the middle term in the expansion of $\left(\frac{x}{3}+9y\right)^{10}$

12. Find the positive value of *m* for which the coefficient of x^2 in the expansion of $(1+x)^m$ is 6.

13. Find the r^{th} term in the expansion of $\left(x+\frac{1}{x}\right)^{2r}$.

14. If p is a real number and if the middle term in the expansion of $\left(\frac{P}{2}+2\right)^{8}$ is 1120, then find the values of p.

15. If n is even and the middle term in the expansion of $\left(x^{2}+\frac{1}{x}\right)^{n}$ is 924 x^6, then find the value of n.

16. Find the co-efficient of x^{11} in the expansion of $\left(x^{3}-\frac{2}{x^{2}}\right)^{12}$.

17. Find the term independent of x in the expansion of $\left(3x-\frac{2}{x^{2}}\right)$

18. Find the expansion of $\left(x^{2}+\frac{3}{x}\right)^{4}$ $x \neq 0$

19. Find the term independent of x in the expansion of $\left(2x-\frac{1}{3x}\right)^{6}$

20. Find the coefficient of x^5 in the expansion of $(x+3)^{6}$.

CHAPTER – SEQUENCE AND SERIES

(1 Mark Questions)

1. 5^{th} term of a G.P. is 2, then the product of first 9 terms is :

(*a*) 256 (*b*) 128 (*c*) 512 (*d*) None of these

2. If a, b, c are in A.P., then : (a + 2b – c) (2b + c – a) (c + a – b) equals :

(*a*) $\frac{abc}{2}$ (*b*) abc (*c*) $2abc$ (*d*) $4abc$

3. Sum of the series is $1^{2}+2^{2}+3^{2}+.........+n^{2}$:

(*a*) $\frac{n}{2}\left(4n^{2}-1\right)$ (*b*) $\frac{n(n+1)(2n+1)}{2}$ (*c*) $\frac{n(n+1)(2n-1)}{2}$ (*d*) $\frac{n(n+1)}{2}$

4. The sum of the first n odd numbers is

(*a*) 2n (*b*) n^2 (*c*) $\frac{n(n-1)}{2}$ (*d*) $\frac{n(n+1)}{2}$

5. If the third term of a G.P. is 3, then the product of its first 5 terms is :

(*a*) 15 (*b*) 81 (*c*) 243 (*d*) Cannot be determined.

6. 5^{th} term of a G.P. is 2, then the product of its 9 terms is :

(*a*) 256 (*b*) 512 (*c*) 1024 (*d*) None of these

7. If the p^{th}, q^{th} and r^{th} terms of G.P. are a, b and c respectively. Then $a^{q-r}\ b^{r-p}\ c^{p-q}$ is equal to

(*a*) 0 (*b*) 1 (*c*) 2 (*d*) -1

8. Find the number of terms between 200 and 400 which are divisible by 7.

(*a*) 28 (*b*) 23 (*c*) 29 (*d*) 27

9. Which term in the A.P. 5,2,-1,...... is -22 ?

(*a*) 10 (*b*) 11 (*c*) 12 (*d*) 9

(4 Marks Questions)

1. Determine 2^{nd} term and r^{th} term of an A.P. whose 6^{th} term is 12 and 8^{th} term is 22.

2. Sum of the first p, q and r terms of an A.P. are a, b and c respectively. Prove that

$$\frac{q}{p}(q-r)+\frac{b}{q}(r-p)+\frac{c}{r}(p-q)=0$$

3. If the 12^{th} term of an A.P. is –13 and the sum of the first four terms is 24, what is the sum of the first 10 terms?

4. Insert 3 A.M's between 3 and 19.

5. The sum of three numbers in A.P. is –3 and their product is 8. Find the numbers.

6. The digits of a positive integer having three digits are in A.P. and their sum is 15. The number obtained by reversing the digits is 594 less than the original number. Find the number.

7. If a^2, b^2, c^2 are in A.P., prove that : $\frac{1}{b+c}, \frac{1}{c+a}, \frac{1}{a+b}$ are also in A.P.

8. Find a G.P. for which sum of the first two terms is –4 and fifth term is 4 times the third term.

9. The value of *n* so that $\frac{a^{n+1}+b^{n+1}}{a^n+b^n}$ may be the geometric mean between *a* and *b*.

10. Determine the number 'n' in a geometric progression $\{a_n\}$, if $a_1 = 3, a_n = 96$ and $s_n = 189$.

11. Sum to n terms : 4 + 44 + 444 + ………

12. Find the sum of 50 terms of a sequence : 7, 7.7, 7.77, 7.777, ……………

13. The arithmetic mean between two numbers is 10 and their geometric mean is 8. Find the numbers.

14. The first term of a G.P. is 2 and the sum to infinity is 6. Find the common ratio.

15. Evaluate : $.23\overline{45}$

16. Find the sum of n terms of the series : $1^2 + 3^2 + 5^2 + \ldots\ldots\ldots\ldots$.

17. Sum to n terms the series : $1^2 - 2^2 + 3^2 - 4^2 + 5^2 - 6^2 + \ldots\ldots\ldots\ldots$.

CHAPTER-STRAIGHT LINES

(1 mark question)

Q.1 Find the distance of the point (4,1) from line 3*x*-4*y*-9=0

(A) $\frac{1}{5}$ (B) $\frac{2}{5}$ (C) $\frac{-1}{5}$ (D) $\frac{-3}{5}$

Q.2 The equation of straight line passing through the point (2,3) and perpendicular to the line $4x - 4y = 10$ is

(A) $-3x + 4y = 15$ (B) $4x + 3y = 5$

(C) $3x + 4y = 18$ (D) $3x + 10y = 4$

Q.3. Find the value of x for which the points (x,-1) (2,1) and (4,5) are collinear.

(A) 2 (B) -1 (C) 1 (D) 0

Q.4. Find the distance between the parallel lines $3x-4y+7=0$ and $3x-4y+5=0$

(A) $\frac{2}{5}$ (B) $\frac{3}{5}$ (C) $\frac{-2}{5}$ (D) $\frac{-3}{5}$

Q.5. Find the angle between the lines $x+y+7=0$ and $x-y+1=0$

(A) 0^0 (B) 45^0 (C) 90^0 (D) 270^0

Q.6. Find the values of k for the line

$(k-3)x+(4-k^2)y+k^2-7k+6=0$ which is parallel to the x-axis

(A) ± 2 (B) 2 (C) -2 (D) 3

Q.7. The lines $a_1n+b_1y+c_1=0$ and $a_2n+b_2y+c_2=0$ are perpendicular to each other if

(A) $a_1b_2=a_2b_1$ (B) $a_1a_2=b_1b_2$

(C) $a_1a_2+b_1b_2=0$ (D) $a_1b_2+a_2b_1=0$

Q.8. Find the equation of a line passing through the point (0,1) and parallel to $3x-2y+5=0$

(A) $x+2y+6=0$ (B) $3x-2y+2=0$

(C) $2x+3y-2=0$ (D) $3x-2y+9=0$

Q.9. Find the slope and y-intercept of st. line $5x+6y=7$

(A) $\frac{-6}{7},\frac{3}{7}$ (B) $\frac{-6}{5},\frac{6}{7}$

(C) $\frac{-5}{6},\frac{7}{6}$ (D) $\frac{-3}{2},\frac{3}{7}$

Q.10. Find the equation of the line perpendicular to the line $2x-3y+7=0$ and having x-intercept is 4.

(A) $3x+2y-12=0$ (B) $3x-2y+6=0$

(C) $-3x+2y+4=0$ (D) $2x-3y-12=0$

Q.11. A line has slope m and y intercept 4, the distance between the origin and the line is equal to

(A) $\frac{4}{\sqrt{1-m^2}}$ (B) $\frac{4}{\sqrt{m^2-1}}$ (C) $\frac{4}{\sqrt{m^2+1}}$ (D) $\frac{4m}{\sqrt{1+m^2}}$

(E) $\frac{4m}{\sqrt{m^2-1}}$

Q.12. Find the distance between st. line $4x+3y-5=0$ and the point $(-2,-1)$

(A) $\frac{16}{5}$ (B) $\frac{9}{4}$ (C) $\frac{-4}{5}$ (D) $\frac{3}{5}$

(4 Marks Questions)

1. Find a point on x axis, which is equidistant from (7, 6) and (3, 4).

2. Show that the points (4, 4), (3, 5) and (–1, –1) are the vertices of a right-triangle.

3. Find the coordinates of the points, which divide internally and externally the line joining (1, –3) and (–3, 9) in the ratio 1 : 3.

4. Find the centroid by the triangle with vertices at (–1, 0), (5, –2) and (8, 2).

5. Find the coordinates of incentre of the triangle whose vertices are (–36, 7); (20, 7) and (0, –8).

6. A point moves so that the sum of its distances from the points (ae, o) and (–ae, o) is 2a. Prove that its locus is : $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ where $b^2=a^2(1-e^2)$

7. State whether the two lines are parallel, perpendicular or neither parallel nor perpendicular:

(*a*) Through (5, 6) and (2, 3); through (9, –2) and (6, –5).

(*b*) Through (2, –5) and (–2, 5); through (6, 3) and (1, 1).

8. Find equation of the line bisecting the segment joining the points (5, 3), (4, 4) and making an angle 45^0 with the x-axis.

9. The perpendicular from the origin to a line meets it at the point (–2, 9), find the equation of the line.

10. Write the equation of the line for which $\tan\theta = \frac{1}{2}$, where θ is the inclination of the line and (i) y-intercept is $\frac{-3}{2}$. (ii) x– intercept is 4.

11. Find the perpendicular form of the equation of the lines from the given values of p and α: (i) p = 3 and $\alpha = 45^0$, (ii) p = 5, $\alpha = 135^0$

12. Find the slope and y– intercept of the straight line $5x + 6y = 7$.

13. Two lines passing through the point (2, 3) make an angle of 45^0. If the slope of one of the lines is 2, find the slope of other.

14. Determine the angle B of the triangle with vertices A(–2, 1), B(2, 3) and C(–2, –4).

15. Find the equation of the straight line through the origin making angle of 60^0 with the straight line $x+\sqrt{3}y+3\sqrt{3}=0$.

16. Find the equation of a line passing through the point (0, 1) and parallel to $3x - 2y + 5 = 0$.

17. If $3x - by + 2 = 0$ and $9x + 3y + a = 0$ represent the same straight line, find the values of 'a' and 'b'.

18. Find the co-ordinates of the orthocentre of the triangle whose angular points are (1,2) (2, 3) and (4, 3).

19. Prove that these lines : $2x - 37 = 7, 3x - 4y = 13$ and $8x - 11y = 33$ meet in a point.

20. Find the equation of the line passing through the point of intersection of $x + 2y = 5$ and $x - 3y = 7$, and passing through the point : (a) (0, –1); (b) (2, –3)

CHAPTER - CONIC SECTION

(1 mark question)

Q.1 If the eq. of the circle is $x^2 + y^2 + 8x - 10y + 8 = 0$ then its centre is

(A) (8,-10), (B) (-8,10), (C) (-4,5) (D) (4,-5)

Q.2. Find the equation of the circle whose centre is (–3, 2) and radius 4.

(A) $x^2 + y^2 + 6x - 4y + 4 = 0$ (B) $x^2 + y^2 + 6x - 4y - 3 = 0$

(C) $x^2 + y^2 - 6x + 4y + 4 = 0$ (D) $x^2 + y^2 - 6x + 4y - 3 = 0$

Q.3 The directrix of the Parabola $y^2 = 4ax$ is

(A) $x = -a$ (B) $x - a = 0$

(C) $x = 0$ (D) None of the these

Q.4 The foci of the ellipse $9x^2 + 4y^2 = 36$ are

(A) $(-5, 0)$ (B) $(0, \pm\sqrt{5})$

(C) $(\pm 5, 0)$ (D) $(0, -5)$

Q.5 The eccentricity of the parabola $y^2 = -8x$ is

(A) -2 (B) 2 (C) -1 (D) 1

Q.6 The eccentricity of the ellipse $x^2 + y^2 + 8y - 2x + 1 = 0$ are

(A) $\frac{\sqrt{3}}{2}$ (B) $\frac{\sqrt{5}}{2}$ (C) $\frac{1}{2}$ (D) $\frac{1}{4}$

Q.7 If in a Hyper-bola, the distance between the foci is 10 and the transverse axis has length 8, than the length of its latus rectum is

(A) 9 (B) $\frac{9}{2}$ (C) $\frac{32}{3}$ (D) $\frac{64}{3}$

Q.8. The focus of the parabola $y^2 = 64x$ is

(A) (16,0) (B) (0,16) (C) (-16,0) (D) (4,0)

Q.9. The eccentricity of circle is

(A) e<1 (B) e > 1 (C) e = 0 (D) $e = \frac{1}{2}$

Q.10. The eccentricity of Hyperbola is

(A) e < 1 (B) e > 1 (C) e = 0 (D) $e = -\frac{1}{2}$

(4 Marks Questions)

1. Find the equation of the circle whose radius is 5 and which touches the circle $x^2 - y^2 - 2x - 4y - 20 = 0$ externally at the point (5, 5).

2. Find the parametric representation of the circle : $x^2 + y^2 - 2x + 4y - 4 = 0$.

3. Show that the point : $x = \frac{2rt}{1+t^2}, y = \frac{r(1-t^2)}{1+t^2}$ (*r* constant) lies on a circle for all values of *t* such that $-1 \leq t \leq 1$.

4. Find the equation of the circle, the co-ordinates of the end-points of whose diameter are (3, 4) and (–3, –4).

5. For the parabola $2y^2 = 5x$, find the vertex, the axis and the focus.

6. Show that the equation $y^2 - 8y - x + 19 = 0$ represents a parabola. Find its vertex, focus and directrix.

7. Find the lengths of the major and minor axes, co-ordinates of the foci, vertices, the eccentricity and equations of the directrices for the ellipse $9x^2 + 16y^2 = 144$

8. Find the equation of the ellipse with $e = \frac{3}{4}$, foci on *y*-axis, centre at the origin, and passing through the point (6, 4).

9. Find the lengths of the transverse and conjugate axes, co-ordinates of the foci, vertices and eccentricity for the hyperbola $9x^2 - 16y^2 = 144$.

10. Find the equation of the parabola satisfying the following conditions :

Vertices at $\left(\pm 0, \frac{\sqrt{11}}{2}\right)$, foci at $(0, \pm 3)$.

CHAPTER – INTRODUCTION TO 3-D GEOMETRY

(2 Marks Questions)

1. Show that the triangle with vertices (6, 10, 10) (1, 0, –5) and (6, –10, 0) is a right angled triangle.

2. Using section formula, prove that (–4, 6, 10) (2, 4, 6) and (14, 0, –2) are collinear.

3. Show that the points A (0, 1, 2), B(2, –1, 3) and C(1, –3, 1) are vertices of right angled isosceles triangle.

4. Show that the points (3, –1, –1), (5, –4, 0), (2, 3, –2) and (0, 6, –3) are vertices of parallelogram.

5. Find the third vertex of triangle whose centroid is (7, –2, 5) and whose other 2 vertices are (2, 6, –4) and (4, –2, 3).

6. Find the point in *XY*-plane which is equidistant from three points A(2,0,3), B(0,3,2) and C(0,0,1) through A.

7. Find lengths of the medians of the triangle with vertices A(0,0,6), B(0,4,0) and C(6,0,0)

8. Find the ratio in which the line joining the points (1,2,3) and (-3,4,-5) is divided by the *XY*-plane. Also, find the co-ordinates of the point of division.

9. Find the ratio in which the plane $3x+4y - 5z = 1$ divides the line joining the points (-2, 4, -6) and (3, -5, 8).

10. Using section formula, show that the points A(2, -3, 4), B(-1, 2, 1) and C(0, 1/3, 2) are collinear.

CHAPTER – LIMIT AND DERIVATIVES

(1 Mark Questions)

1. $\lim_{x \to 0} |x|$ is

(A) 2 (B) 0 (C) does not exist (D) none of these

2. The value of $\lim_{x \to 0} \frac{\sin bx}{\sin ax}$ is equal to

(A) 1 (B) 0 (C) b/a (D) a/b

3. $\lim_{\theta \to 0} \frac{\sin 5\theta}{\theta}$ is

(A) 5 (B) 1/5 (C) 1 (D) none of these

4. $\lim_{x \to 0} \frac{|x|}{x}$ is

(A) 1 (B) -1 (C) 0 (D) Does not exist

5. The value of the derivatives of $h(x) = 4x^4$ at $x = 1/3$ and $x = \frac{-1}{3}$ are

(A) Different (B) Same (C) Negative (D) Positive

6. $\lim_{x\to 0} \sin\frac{ax}{bx}$ is

(A) b/a (B) a/b (C) $\frac{a^2}{b^2}$ (D) $\frac{b^2}{a^2}$

7. The derivative of $\sin x \cos x$ w.r.t x is

(A) $\sin 2x$ (B) $\cos 2x$ (C) $2\sin 2x$ (D) $2\cos 2x$

8. The derivative of $\tan\left(\frac{\pi}{2}-x\right)$ is equation to

(A) $\sec^2\left(\frac{\pi}{2}-x\right)$ (B) $-\operatorname{cosec}^2 x$

(C) $\operatorname{cosec}^2 x$ (D) None of these

9. $\lim_{x\to 0} \frac{\sin 4x}{\sin 2x}$ is :

(A) 2 (B) 1 (C) 4 (D) 3

10. $\lim_{x\to\frac{\pi}{2}} \frac{\sin x}{x}$ is

(A) $\frac{\pi}{2}$ (B) $\frac{2}{\pi}$ (C) 1 (D) None of these

11. The value of $\lim_{x\to 0} \sin\frac{1}{x}$ is

(A) zero (B) 2 (C) ∞ (D) Does not exist

(4 Mark Questions)

1. Evaluate

(a) $\lim_{x\to 0} \frac{1-\cos x}{x}$ (b) $\lim_{x\to 0} \frac{a^x - 1}{b^x - 1}$

2. Evaluate using factor method :

(a) $\lim_{x \to 1} \frac{x^2 - 1}{x - 1}$ (b) $\lim_{x \to 1/2} \frac{4x^2 - 1}{2x - 1}$

3. Find the derivative of the function :

$f(x)=2x^2+3x-5$ at $x= -1$. Also prove that $f'(0)+3f'(-1)=0$

4. For each of the following functions, evaluate the derivative at the indicated value (s) :

(a) $s = 4.9\,t^2$; $t=1, t=5$ (b) $s = 4x^8$; $x = \frac{-1}{2}, x = \frac{1}{2}$

5. Evaluate $\underset{x \to 0}{Lt} f(x)$ where $f(x) = \begin{cases} \frac{|x|}{x}, x \neq 0 \\ 0, x = 0 \end{cases}$

6. Find $\frac{dy}{dx}$, when $y = \sqrt{3x+2} + \frac{1}{\sqrt{2x^2+4}}$

7. Find $\frac{dy}{dx}$, when $y = (3x^2+2)^3(5x-1)^2$

8. Find $\frac{dy}{dx}$, when $y = \frac{x^2+3}{(2x+1)^2}$

9. Find $\frac{dy}{dx}$, when $y = \sin^2 x.\cos(x^3)$

(6 Mark Questions)

1. Evaluate :

(a) $\lim_{x \to 2} \frac{x^{10} - 1024}{x^5 - 32}$ (b) $\lim_{h \to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h}$

2. Evaluate :

$$\lim_{h \to 0} \frac{1}{h}\left(\frac{1}{\sqrt{x+h}} - \frac{1}{\sqrt{x}}\right)$$

3. Find $\lim_{x \to 1} f(x)$, where $f(x) = \begin{Bmatrix} x^2 - 1, & x \leq 1 \\ -x^2 - 1, & x > 1 \end{Bmatrix}$

4. Evaluate :

(a) $\lim_{x\to 0} \frac{1-\cos 3x}{x^2}$ (b) $\lim_{x\to 0}\left(\frac{1-\cos x}{\sin^2 x}\right)$

5. Evaluate :

$$\lim_{x\to 0} \frac{1-\cos x \cos 2x \cos 3x}{\sin^2 x}$$

6. Evaluate :

$$\underset{x\to 0}{\text{Lt}} \frac{ax + x\cos x}{b\sin x}$$

7. Prove :

$$\lim_{x\to 0} \frac{\log\left(1+x^3\right)}{\sin^3 x} = 1$$

8. Evaluate :

$$\lim_{x\to 0} \frac{e^x - 1}{\sqrt{1-\cos x}}$$

9. Given $f(x) = \frac{1}{\sqrt{x}}, x > 0,$ find $f'(x)$ by delta method.

10. Given $f(x) = x\sin x$, find $f'(x)$ by delta Method

CHAPTER – MATHEMATICAL REASONING

(2 Marks Questions)

1. Write the negation of the following statements:
   a) Both the diagonals of the rectangle have same length.
   b) $\sqrt{7}$ is rational
2. Identify the quantifies in the following statement and write the negation of the statements.
   i) There exists a number, which is equal to its square.
   ii) For every real number x, x is less than $x+1$.
3. Write the converse of the following statements:
   i) If a number x is odd, then x^2 is odd.

ii) If two integers a and b are such that a > b, then (a-b) is always a +ve integer.

4. Let p : He is rich and q: He is happy be the given statements, write each of the following statements in the symbolic form, using p and q.

   i) If he is rich, then he is unhappy.

   ii) It is necessary to be poor is order to be happy.

5. Determine the truth value of the following :

   i) 5+4=9 iff 8-2=6

   ii) Apple is a fruit iff Delhi is in Japan.

6. Show that the following statement is true by the method of contrapositive

   p : if x is an integer and x^2 is even, then x is also even.

7. Verify by the method of contraction :

   $p : \sqrt{7}$ is irrational

8. Given below the two statements :

   p : 25 is a multiple of 5

   q : 25 is a multiple of 8.

   Connecting, these two statements with 'And' and 'Or'. In both cases check the validity of the compound statement.

9. Which of the following are statements and which are not? Give reasons for your answers.

   (i) The number 6 has three prime factors

   (ii) Rajendra Prasad was the first President of India.

10. Write the negation of the following statements.

    (i) The number 2 is greater than 7.

    (ii) All triangles are not equilateral triangles.

11. Write the negation of following statement.

    (i) Australia is a continent.

    (ii) Every natural number is greater than 0.

12. Find the component statements of the following compound statements.

    (i) 25 is a multiple of 5 and 8.

(ii) The sun shines or it rains.

13. Find the component statements of the following and check whether they are true or not.

(i) All prime numbers are either even or odd.

(ii) India is a democracy and a monarchy.

14. Write each of the statements in the form 'if P, then q'

(i) P : It is necessary to have a password in log on to the server.

(ii) q : There is traffic jam whenever it rains

15. Write the contra positive of the following statements

(i) If a number is divisible by 9, then it is divisible by 3.

(ii) If you are born in India, then you are a citizen of India.

16. By giving a counter example, show that the following statements are not true.

(i) If n is an odd integer, then n is prime.

(ii) The equation $x^2 - 4 = 0$ does not have a root lying between 0 and 3.

17. Show by the method of contradiction $P: \sqrt{2}$ is irrational.

18. Show that the statement

"Given a positive number x, there exists a rational number r such that $0 < r < x^3$ is true

19. Determine the truth value of each of the following statements.

(i) 3+3=6 iff 2+2=4

(ii) 3+3 = 7 iff 5+2=6

20. Given below are pairs of statements combine them using 'if and only if'

(i) P : If two lines are parallel, then their slopes are equal

q : if the slopes of two lines are equal, then they are parallel

CHAPTER - STATISTICS

(6 Marks questions)

Q.1 If $\bar{x}$ is the mean and Mean Deviation from mean is MD($\bar{x}$), then find the number of observations lying between $\bar{x}$-MD($\bar{x}$) and $\bar{x}$+MD($\bar{x}$) from the following data : 22, 24, 30, 27, 29, 31, 25, 28, 41, 42.

Q.2 Calculate the mean deviation about median for the following data.

Class	0-10	10-20	20-30	30-40	40-50	50-60
Frequency	6	7	15	16	4	2

Q.3 Calculate the mean, variance and standard deviation for the following distribution :

Class	30-40	40-50	50-60	60-70	70-80	80-90	90-100
Frequency	3	7	12	15	8	3	2

Q.4 The mean and variance of 8 observations are 9 and 9.25 respectively. If six observations are 6,7,10,12,12,13, find the remaining two observations.

Q.5 Calculate the mean and variance for the following data :

Income (in Rs.)	1000-1700	1700-2400	2400-3100	3100-3800	3800-4500	4500-5200
No. of families	12	18	20	25	35	10

Q.6 Find the mean and variance for the data.

x_i	6	10	14	18	24	28	30
y_i	2	4	7	12	8	4	3

CHAPTER – PROBABILITY

(One mark questions)

1. In a single through of two dice, the probability of getting a total other than 9 or 11 is :

(*a*) $\frac{1}{6}$ (*b*) $\frac{1}{9}$ (*c*) $\frac{1}{18}$ (*d*) $\frac{5}{18}$

2. Two numbers are chosen from {1, 2, 3, 4, 5, 6} one after another without replacement. Find the probability that one of the smaller value of two is less than 4 :

(*a*) $\frac{4}{5}$ (*b*) $\frac{1}{15}$ (*c*) $\frac{3}{5}$ (*d*) $\frac{14}{15}$

3. Three houses are available in a locality. Three persons apply for the houses. Each applies for one house without consulting the other. The probability that all 3 apply for the same house is :

(*a*) $\frac{1}{9}$ (*b*) $\frac{2}{9}$ (*c*) $\frac{7}{9}$ (*d*) $\frac{8}{9}$

4. If 3 distinct numbers are chosen randomly from the first 100 natural numbers then the probability that all 3 of them are divisible by 2 and 3 is :

(*a*) $\frac{4}{25}$ (*b*) $\frac{4}{35}$ (*c*) $\frac{4}{33}$ (*d*) $\frac{4}{1155}$

5. What is the chance that a leap year, selected at random, will contain 53 Sundays?

(*a*) $\frac{1}{7}$ (*b*) $\frac{3}{7}$ (*c*) $\frac{2}{7}$ (*d*) $\frac{5}{7}$

6. Find the probability that in a random arrangement at the word 'Society' all the three vowels come together.

(*a*) $\frac{4}{7}$ (*b*) $\frac{3}{7}$ (*c*) $\frac{1}{7}$ (*d*) $\frac{5}{7}$

7. One card is drawn from a well shuffled deck of 52 cards. If each outcome is equally likely, calculate the probability that the card will be a diamond :

(*a*) $\frac{1}{4}$ (*b*) $\frac{1}{2}$ (*c*) $\frac{3}{4}$ (*d*) $\frac{1}{6}$

8. In a single throw of three dice, find the probability of getting a total of atmost 5.

(*a*) $\frac{7}{108}$ (*b*) $\frac{5}{108}$ (*c*) $\frac{1}{108}$ (*d*) $\frac{1}{216}$

9. From an urn containing 2 white and 6 green balls, a ball is drawn at random. The probability of not a green ball is :

(*a*) $\frac{1}{4}$ (*b*) $\frac{3}{4}$ (*c*) $\frac{1}{3}$ (*d*) $\frac{2}{3}$

(4 marks questions)

1. A letter is chosen at random from the ward 'ASSASSINATION' . Find the probability that letter is

(i) a vowel (ii) a consonant.

2. A coin is tossed three times. Consider the following events :

A : No head appears.

B : Exactly one head appears.

C : At least two heads appear.

Do they form a set of mutually exclusive and exhaustive events?

3. Two dice are thrown and the sums of numbers which come up on the dice are noted. Consider the following events :

A : the sum is even.

B : the sum is a multiple of 3.

C : the sum is less than 4.

D : the sum is greater than 11.

4. A die is thrown, find the probability of the following events :

(*a*) A prime number will appear (*b*) A number less than 6 will appear

5. One card is drawn from a well shuffled deck of 52 cards. If each outcome is equally likely, calculate the probability that the card will be :

(*a*) a diamond (*b*) not an ace (*c*) a black card (*d*) not a black card

6. In a throw of 2 coins, find the probability of getting both heads or both tails.

7. A bag contains 8 red, 3 white and 9 blue balls. Three balls are drawn at random from the bag. Determine the probability that none of the balls is white.

8. Find the probability of 4 turning for at least once in two tosses of a fair die.

9. A and B are two mutually exclusive events, for which P(A) = 0.3, P(B) = p and P(AUB) = 0.5. Find 'p'.

10. In a class of 25 students with roll numbers 1 to 25, a student is picked up at random to answer a question. Find the probability that the roll number of the selected student is either a multiple of 5 or 7.

SAMPLE PAPER – I

CLASS – XI

MATHEMATICS

Time : 3 hrs. Theory : 90 marks

CCE : 10 marks

Total : 100 marks

1. All questions are compulsory.
2. Q.1. will consist of 10 parts and each part will carry one [1] marks.
3. Q.2 to Q. 9 each will be of 2 marks.
4. Q.10 to Q. 19 each will be of 4 marks.
5. Q.20 to Q. 23 each will be of 6 marks.
6. There will be no overall choice. There will be an internal choice in any 3 questions of 4 marks each and all questions of 6 marks [Total of 7 internal choices]
7. Use of calculator is not allowed.

Q.1.(i) If $S = \{0,1,5,4,7\}$ then the total number of subsets of S is equal to : (1)

(a) 64 (b) 32 (c) 140 (d) 20

(ii) Let $f(x) = [x]$ then $f\left(\frac{-3}{2}\right)$ is equal to (1)

(a) -3 (b) -2 (c) -1.5 (d) none of these

(iii) Value of $\sin 765^0$ is (1)

(a) 1 (b) $\sqrt{2}$ (c) $\frac{1}{\sqrt{2}}$ (d) 765^0

(iv) $(5-3i)^3$ in the form of $a+ib$ can be written as (1)

(a) $10-198i$ (b) $5-3i$ (c) $(5-3i)^2$ (d) $3-5i$

(v) If $\frac{1}{8!}+\frac{1}{9!}=\frac{x}{10!}$ then the value of x will be (1)

(a) 10 (b) 100 (c) 8 (d) 9

(vi) The 20th term of the sequence defined by (1)

$a_n = (n-1)(2-n)(3+n)$ is equal to

(a) – 6768 (b) -6678 (c) 6678 (d) -7866

(vii) The slope of the line passing through the points (3,-2) and (3,4) is (1)

(a) $\frac{-3}{2}$ (b) not defined (c) 0 (d) $\sqrt{3}$

(viii) The equation of the circle with centre (-3,2) and radius 4 is (1)

(a) $(x+3)^2 + (y-2)^2 = 16$ (b) $(x-2)^2 + (y+7)^2 = 4$

(c) $(x-2)^2 + (y+3)^2 = 16$ (d) $x^2 + y^2 = y$

(ix) The value of $\lim_{x \to 1} \left[\frac{x^2 + 1}{x + 100} \right]$ is equal to (1)

(a) 1 (b) $\frac{201}{10}$ (c) $\frac{101}{2}$ (d) $\frac{2}{101}$

(x) Two coins (a one rupee coin and a two rupee coin) are tossed once the sample space will be (1)

(a) {HH,HT,TH,TT} (b) {HH, TT}

(c) {HT, TH} (d) {HH, HT, TH}

Q.2. Prove that $\sec^4 A - \sec^2 A = \tan^4 A + \tan^2 A$ (2)

Q.3. Write down the values of $\cos 68^0 \cos 8^0 + \sin 68^0 . \sin 8^0$ (2)

Q.4. If $x + iy = \frac{a+ib}{a-ib}$ prove that $x^2 + y^2 = 1$ (2)

Q.5. Expand $\left(\frac{x}{3} + \frac{1}{x} \right)^5$ $x \neq 0$ (2)

Q.6. Find the middle term in the expansion of $\left(3 - \frac{x^3}{6} \right)^7$? (2)

Q.7. Show that the points P(-2, 3, 5) Q (1,2,3) and R (7,0,-1) are collinear. (2)

Q.8. Write the negation of the following statement

(a) Both the diagonals of the rectangle have same length. (2)

(b) $\sqrt{7}$ is rational

Q.9. Find the component statements of the following compound statements. (2)

(i) 25 is a multiple of 5 and 8.

(ii) The sun Shines or it rains

Q.10. If $A = \{2,3\}$, $B = \{x : x$ is a root of $x^2 + 5x + 6 = 0\}$ then find (4)

(i) $A \cup B$

(ii) $A \cap B$

(iii) Are they equal sets?

(iv) Are they equivalent sets?

Q.11. If G = {7,8 } and H = {5,4,2} find G×H and H×G (4)

Q.12. Prove that $\sin\frac{\pi}{5}\sin\frac{2\pi}{5}\sin\frac{3\pi}{5}\sin\frac{4\pi}{5} = \frac{5}{16}$ (4)

Q.13. By principle of mathematical Induction, prove that (4)

$$1^2 + 2^2 + 3^2 + ... + n^2 = \frac{1}{6}\, n(n+1)(2n+1)$$

Q.14. Find n such that $\frac{{}^{n}P_4}{{}^{n-1}P_4} = \frac{5}{3}$, $n > 4$ (4)

Q.15. Determine 2nd term and rth term of an A.P. whose 6th term is 12 and 8th (4) term is 22.

OR

How many triangles can be obtained by joining 12 Points, 5 of which are collinear?

Q.16. The perpendicular from the origin to a line meets it at the point (-2, 9), find the equation of the line. (4)

Q.17. Find the equation of the ellipse whose vertices are ($\pm$ 13,0) and foci are ($\pm$ 5,0)

OR (4)

Find the equation of the circle whose radius is 5 and which touches the circle $x^2 - y^2 - 2x - 4y - 20 = 0$ externally at the point (5,5)

Q.18. Evaluate using factor method (4)

(a) $\lim_{x\to 1}\frac{x^2-1}{x-1}$ (b) $\lim_{x\to \frac{1}{2}}\frac{4x^2-1}{2x-1}$

Q.19. A coin is tossed three times. Consider the following events (4)

A : No head appears

B : Exactly one head appears

C : At least two heads appear

Do they form a set of mutually exclusive and exhaustive events?

OR

A and B are two mutally exclusive events, for which P(A)= 0.3, P(B)=P and $P(A \cup B) = 0.5$ find 'P'.

Q.20. Convert into polar form $\frac{1+7i}{(2-i)^2}$ (6)

OR

Solve : $x^2-\left(3\sqrt{2}+2i\right)x+6\sqrt{2}i=0$

Q.21. Solve the following in equations and show the graph on number line. (6)

(a) $3x-6<0$ (b) $-3x+9\le 0$

(c) $7x+5>33$ (d) $5x-15\ge 0$

OR

Solve the following system of in equations.

$\frac{5x}{4}+\frac{3x}{8}>\frac{39}{8}$ and $\frac{2x-1}{12}-\frac{x-11}{3}<\frac{3x+1}{4}$

Q.22. Evaluate $\lim_{h\to 0}\frac{1}{h}\left(\frac{1}{\sqrt{x+h}}-\frac{1}{\sqrt{x}}\right)$ (6)

OR

Find $\lim_{x\to 1} f(x)$, where $f(x)=\begin{cases} x^2-1, & x\le 1 \\ -x^2-1, & x>1 \end{cases}$

Q.23. The mean and variance of 8 observations are 9 and 9.25 respectively. If six observations are 6, 7, 10, 12, 12, 13 find the remaining two observations. (6)

OR

Calculate the mean deviation about median for the following data.

Class	0-10	10-20	20-30	30-40	40-50	50-60
Frequency	6	7	15	16	4	2

SAMPLE PAPER – II

CLASS – XI

MATHEMATICS

Time : 3 hrs. Theory : 90 marks

CCE : 10 marks

Total : 100 marks

1. All questions are compulsory.
2. Q.1. will consist of 10 parts and each part will carry one [1] marks.
3. Q.2 to Q. 9 each will be of 2 marks.
4. Q.10 to Q. 19 each will be of 4 marks.
5. Q.20 to Q. 23 each will be of 6 marks.
6. There will be no overall choice. There will be an internal choice in any 3 questions of 4 marks each and all questions of 6 marks [Total of 7 internal choices]
7. Use of calculator is not allowed.

Q.1.(i) $(A \cup B)^C$ is equal to (1)

(a) $A^C \cup B^C$ (b) $A^C \cap B^C$

(c) $A^C - B^C$ (d) None of these

(ii) Let $f(x) = [x]$, then $f\left(\frac{-3}{2}\right)$ is equal to (1)

(a) -3, (b) -2, (c) -1.5 (d) None of these

(iii) Value of sin 585^0 is (1)

(a) 1 (b) $\frac{1}{\sqrt{2}}$ (c) $\frac{-1}{\sqrt{2}}$ (d) 2

(iv) Complex conjugate of 3i-4 is (1)

(a) 3i+4, (b) -3i-4 (c) -3i+4, (d) None of these

(v) The value of $15_{C_{11}} \div {}^{15}C_{10}$ is (1)

(a) $\frac{15}{11}$ (b) $\frac{15}{10}$ (c) $\frac{5}{11}$ (d) $\frac{5}{10}$

(vi) Which term in the A.P. 5, 2, -1 .... is -22? (1)

(a) 10 (b) 11 (c) 12 (d) 9

(vii) Find the distance, of the point (4,1) from line $3x$-$4y$-9=0 (1)

(a) $\frac{1}{5}$ (b) $\frac{2}{5}$ (c) $\frac{-1}{5}$ (d) $\frac{-3}{5}$

(viii) The eccentricity of circle is (1)

(a) e<1 (b) e>1 (c) e=0 (d) e=1/2

(ix) $\lim_{x\to 0}\frac{|x|}{x}$ is

(a) 1 (b) -1 (c) 0 (d) Does not exist.

(x) In a single through of two dice, the probability of getting a total sum 11 is

(a) $\frac{1}{36}$ (b) $\frac{1}{12}$ (c) $\frac{1}{18}$ (d) $\frac{1}{9}$

Q.2. Prove that $\sqrt{\frac{1+\sin\theta}{1-\sin\theta}} = \sec\theta + \tan\theta$ (2)

Q.3. Prove that $\sin 70^0 \cos 10^0 - \cos 70^0 \sin 10^0 = \frac{\sqrt{3}}{2}$ (2)

Q.4. Solve the equation. (2)

$x^2 - 7ix - 12 = 0$

Q.5. Write the 4^{th} term in the expansion of $\left(3-\frac{x^3}{6}\right)^7, x>0$ (2)

Q.6. Find the coefficient of x^5 in the expansion of $(x+3)^6$. (2)

Q.7. Show that the points A (0,1,2), B (2,-1,3) and C (1,-3,1) are vertices of right angles isosceles triangle. (2)

Q.8. Write the negative of following statements. (2)

(i) Australia is a continents.

(ii) Every natural number is greater than zero.

Q.9. Determine the truth value of each of the following statements. (2)

(i) 3+3=6 off 2+2=4

(ii) 3+3=7 off 5+2=6

Q.10. Let U={1,2,3,4,5,6,7,8,9}, A={1,2,3,4}, B={2,4,6,8}. Find (4)

(a) A^C (b) B^C (c) $(A^C)^C$ (d) $(A \cup B)^C$

Q.11. Let A = {1,2,3}, B={2,3,4}, C={4,5} verify that (4)

$$A\times(B\cap C)=(A\times B)\cap(A\times C)$$

Q.12. Show that (4)

$$\sqrt{2+\sqrt{2+\sqrt{2+2\cos\theta}}} = 2\cos\theta$$

Q.13. By Principle of Mathematical Induction, prove that $7^n - 3^n$ is divisible by 4, for all $n \in N$ (4)

Q.14. In how many ways can 5 persons- A, B, C, D and E sit around a circular table if (4)

(a) B and D sit next to each other.

(b) A and D do not sit next to each other.

Q.15. The sum of three numbers in A.P. is -3 and their product is 8. Find the numbers.

Or

Prove that $\tan 9^0 - \tan 27^0 - \tan 63^o + \tan 81^o = 4$

Q.16. Find the equation of a line passing through the point (0,1) and parallel to $3x-2y+5=0$ (4)

Q.17. For the parabola $2y^2 = 5x$, Find the vertex, the axis and the focus. (4)

OR

Find the centroid by the triangle with vertices at (-1,0), (5,-2) and (8,2)

Q.18. Find $\frac{dy}{dx}$, when $y = (3x^2+2)^3 (5x-1)^2$ (4)

Q.19. A die is thrown, find the probability of the following events. (4)

(a) A prime number will appear.

(b) A number less than 6 will appear.

OR

Evaluate $\underset{x \to 0}{Lt} f(x)$ where $f(x) = \begin{Bmatrix} \frac{|x|}{x}, & x \neq 0 \\ 0, & x = 0 \end{Bmatrix}$

Q.20. If $(x+iy)^3 = u+iv$, then show that : (6)

$$\frac{u}{x} + \frac{v}{y} = 4(x^2 - y^2)$$

OR

Convert into polar form : $z = \frac{i-1}{\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}}$

21. Find the region enclosed by the following in equations (6)

$x + y - 2 \le 0,\ 2x + y - 3 \le 0,\ x \ge 0,\ y \ge 0$

OR

Solve the following system of inequations :

$2(2x+3) - 10 < 6(x-2)$ and $\frac{2x-3}{4} + 6 \ge 2 + \frac{4x}{3}$

22. $\underset{x \to 0}{Lt} \frac{\log(1+x^3)}{\sin^3 x} = 1$ (6)

OR

Given $f(x) = x\sin x$, Find $f'(x)$ by delta method.

23. Calculate the mean, variance and standard deviation for the following distribution:

(6)

Class	30-40	40-50	50-60	60-70	70-80	80-90	90-100
Frequency	3	7	12	15	8	3	2

OR

Find the mean and variance for the data :

x_i	6	10	14	18	24	28	30
y_i	2	4	7	12	8	4	3

CLASS - 10+2

MATHEMATICS

CONTENTS

CHAPTER 1

RELATIONS AND FUNCTIONS

(1 Mark Questions)

1) If number of elements in set A and B are m and n respectively, then the number of relations from A to B is

(a) 2^{m+n} (b) 2^{mn} (c) m+n (d) mn

2) Let A = {1,2,3,4} and Let R={(2,2), (3,3), (4,4), (1,2)} be relation in A, then R is

(a) Reflexive (b) Symmetric (c) Transitive

(d) None of these.

3) Let A={a,b,c} and B={1,2}. Consider a relation R defined from Set A to set B. Then, R is equal to subset of

(a) A (b) B (c) A x B (d) B x A

4) Let A= {1,2,3}. The total number of distinct relations that can be defined over A is

(a) 2^9 (b) 6 (c) 8 (d) None of these

5) R is a relation on N given by N = {(x,y): $4x+3y=20$}. Which of the following belongs to R?

(a) (-4, 12) (b) (5, 0) (c) (3,4) (d) (2,4)

6) Let X be a family of sets and R be a relation in X, defined by 'A is disjoint from B'. Then, R is

(a) Reflexive (b) Symmetric (c) Anti-Symmetric (d) Transitive.

7) For an onto function f: {1,2,3} $\rightarrow$ {1,2,3} is always

(a) into (b) one-one (c) not one-one (d) Many one

8) Function f: R $\rightarrow$ R defined by f(x)=x^2 is

(a) one-one (b) onto (c) one-one onto

(d) Neither one-one nor onto.

9) The function f: R $\rightarrow$ R defined by f(x) = cos x is

(a) into (b) onto (c) one-one (d) many-one onto

10) If f: R $\rightarrow$R is defined by f(x) = $(3 - x^2)^{1/3}$, then fof (x) is

(a) x^3 (b) $x^{1/3}$ (c) x (d) 9

11) Number of all one-one functions from Set A={1,2,3} to itself is

(a) 3 (b) 6, (c) 8 (d) 9

12) If f: A $\rightarrow$B and g: B $\rightarrow$C are onto then gof : A $\rightarrow$C is

(a) onto (b) one-one (c) not onto

(d) one one but not onto

13) Function f: x $\rightarrow$y is invertible it

(a) f is one one (b) f is onto

(c) f is one-one onto (d) f is one-one but not onto.

14) Let a * b = 2a+b, '*' be a binary opration, then 3*4 equals

(a) 7 (b) 9 (c) 10 (d) None of these

15) If, f(x) = $x^2 - 1$, and g(x) = $\sqrt{x}$, then gof (1) is

(a) -1 (b) 0 (c) 1 (d) 2

(4 Mark Questions)

1) Check the following functions for one-one and onto.

(a) $f : R \rightarrow R,\ f(x) = \frac{2x-3}{7}$

(c) $f : R \rightarrow R, f(x) = |x+1|$

2) Prove that the Greatest integer function $f : R \rightarrow R$,given by f(X)= $[x]$,is neither one-one nor onto, where $[x]$ denotes the Greatest integer less than or equal to x

3) **Consider $y: R^+ \to [4, \infty)$ given by $y = x^2 + 4$. Show that *f* is both one-one and onto, where R^+ is the set of all non-negative real numbers. Express *x* in terms of *y*.**

4) Check the function for one-one and onto f: R $\to$ R, $f(x)=9x^3$

5) Consider f : R $\to$ R given by f(*x*) = 4*x*+3. Show that *f* is invertible. Find inverse of *f*.

6) Let A=R-{3}and B=R-{1},consider the function f : A $\to$ B defined by

$f(x) = \frac{x-1}{x-3}$ show that f is one-one and onto and hence find f^{-1}

7) Show that the modulus function $f: R \to R$ defined by f(x) = | 2x | is neither one-one nor onto.

8) Check the function $f: R \to R$ given by $f(x) = x^3 - 6x^2 + 11x - 6$ is one-one or not.

9) Let $f: X \to Y$ and $g: Y \to Z$ be two invertible function, then show that $(gof)^{-1} = f^{-1}og^{-1}$.

10) If $f: X \to Y$ and $g: Y \to Z$ are onto functions, then show that g of $X \to Z$ is also onto.

11) If L is the set of all lines in the plane and R is the relation in L defined by R = {(l_1, l_2) : l_1 is parallel to l_2}. Show that the relation R is equivalence relation.

12) Show that the relation R, defined in a set A of all triangles as {(T_1, T_2) : T_1 is similar triangle to T_2}, is equivalence relation.

13) Show that the relation Q in R defined as Q = {(a, b) : b $\geq$ a}, is reflexive and transitive but not symmetric.

14) Show that the relation R in the set A = {a, b, c} given by R = {(b, c), (c, b)} is symmetric but neither reflexive nor transitive.

15) State the reason for the relation R in the set {1, 2, **3} given by R = {(1, 2), (2, 1)} not to be transitive**

16) Let * be a binary operation on Q defined by

$$a * b = \frac{3ab}{2}$$

Show that □ is commutative as well as associative. Also find its identity element, if it exists.

17) Consider the binary operation * on N defined by a * b = LCM (a, b). for all a, b belongs to N. Write the multiplication table for binary operation *. Also find 5 * 7.

18) Consider the binary operation * on the set {1, 2, 3, 4, 5} defined by a * b = min (a, b). Write the multiplication table for binary operation *. Also find (2 * 3) * (4 * 5).

19). If A = N × N and binary operation * is defined on A as (a, b) * (c, d) = (ac, bd).

(i) Check * for commutativity and associativity.

(ii) Find the identity element for * in A (If exists).

CHAPTER-2

INVERSE TRIGONOMETRIC FUNCTIONS

(1 Mark Questions)

1) The principal value of $\sin^{-1}\left(\frac{-\sqrt{3}}{2}\right)$ is

(A) $\left(\frac{-2\pi}{3}\right)$ (B) $\left(\frac{-\pi}{3}\right)$ (C) $\left(\frac{4\pi}{3}\right)$ (D) $\left(\frac{5\pi}{3}\right)$

2) If $\sin^{-1}\left(\frac{3}{5}\right)+\cos^{-1}\left(\frac{12}{13}\right)=\sin^{-1}C$, then C is

(A) $\frac{65}{66}$ (B) $\frac{24}{65}$ (C) $\frac{16}{65}$ (D) $\frac{56}{65}$

3) If $A=\tan^{-1}x$, then the value of $\sin 2A$ is

(A) $\frac{2x}{1-x^2}$ (B) $\frac{x}{1-x^2}$ (C) $\frac{2x}{1+x^2}$ (D) None of these

4) The value of $\cot^{-1}\left[\frac{\sqrt{1-\sin x}+\sqrt{1+\sin x}}{\sqrt{1-\sin x}-\sqrt{1+\sin x}}\right]$ is

(A) $\pi - x$ (B) $2\pi - x$ (C) $\frac{x}{2}$ (D) $\pi-\frac{x}{2}$

5) If $\sin^{-1}\left(\frac{2a}{1+a^2}\right)+\sin^{-1}\left(\frac{2b}{1+b^2}\right)=2\tan^{-1}x$, then *x* equals

(A) $\frac{a-b}{a+ab}$ (B) $\frac{b}{1+ab}$ (C) $\frac{b}{1-ab}$ (D) $\frac{a+b}{1-ab}$

6) The value of $\tan\left[\cos^{-1}\left(\frac{4}{5}\right)+\tan^{-1}\left(\frac{2}{3}\right)\right]$ is

(A) $\frac{6}{17}$ (B) $\frac{16}{7}$ (C) $\frac{7}{16}$ (D) None of these

7) $\sin\left(\frac{\pi}{3}-\sin^{-1}\left(\frac{-1}{2}\right)\right)$ is equal to

(A) $\frac{1}{2}$ (B) $\frac{1}{3}$ (C) $\frac{1}{4}$ (D) 1

8) $\tan^{-1}(\sqrt{3})-\cot^{-1}(-\sqrt{3})$ is equal to

(A) π (B) $\frac{-\pi}{2}$ (C) 0 (D) $2\sqrt{3}$

9) $\tan^{-1}\sqrt{3}-\sec^{-1}(-2)$ is equal to

(A) π (B) $\frac{-\pi}{3}$ (C) $\frac{\pi}{3}$ (D) $\frac{2\pi}{3}$

10. The value of $\tan^{-1}\left[2\cos\left(2\sin^{-1}\left(\frac{1}{2}\right)\right)\right]$ is

(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{3}$ (D) $\frac{2\pi}{3}$

11. The number of real solution of $\tan^{-1}\sqrt{x(x+1)}+\sin^{-1}\sqrt{x^2+x+1}=\frac{\pi}{2}$ is

(A) zero (B) one (C) both (D) infinite

12. The value of *x* which satisfies the equation

$$\tan^{-1}x=\sin^{-1}\left(\frac{3}{\sqrt{10}}\right)$$ is

(A) 3 (B) -3 (C) $\frac{1}{3}$ (D) $\frac{-1}{3}$

13. $\sin^{-1}(1-x)-2\sin^{-1}(x)=\frac{\pi}{2}$ then *x* is equal to

(A) $0,\frac{1}{2}$ (B) $1,\frac{1}{2}$ (C) 0 (D) $\frac{1}{2}$

14. The value of $\tan\left[2\sin^{-1}\left(\frac{4}{5}\right)\right]$ is

(A) $\frac{7}{24}$ (B) $\frac{-7}{24}$ (C) $\frac{-24}{7}$ (D) $\frac{24}{7}$

(4 Mark Questions)

Question 1

1). Find the Principal values of following inverse trigonometric functions :

(i) $\tan^{-1}\left(-\sqrt{3}\right)$ (ii) $\cos^{-1}\left(-\frac{1}{2}\right)$

(iii) $\sin^{-1}\left(-\frac{1}{\sqrt{2}}\right)$ (iv) $\cot^{-1}\left(-\frac{1}{\sqrt{3}}\right)$

Question2 . **Prove the following :**

$$\tan^{-1}\sqrt{x} = \frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right), x \in (0,1)$$

Question 3 Prove the following :

$$\cos^{-1}\left(\frac{12}{13}\right) + \sin^{-1}\left(\frac{3}{5}\right) = \sin^{-1}\left(\frac{56}{65}\right)$$

Question 4 Write the principal value of $\cos^{-1}\left(\frac{1}{2}\right) - 2\sin^{-1}\left(-\frac{1}{2}\right)$

Question 5 ***Prove that :*** $\tan^{-1} 2 + \tan^{-1} 3 = \frac{3\pi}{4}$

Question 6 ***Prove that*** $\tan^{-1}1 + \tan^{-1}2 + \tan^{-1}3 = \pi$

Question 7 ***Solve for*** x $2\tan^{-1}x = \sin^{-1}\left(\frac{2\alpha}{1+\alpha^2}\right) - \cos^{-1}\left(\frac{1-\beta^2}{1+\beta^2}\right)$

Question 8 ***Prove that*** $\tan^{-1}x + \cot^{-1}(x+1) = \tan^{-1}\left(x^2+x+1\right)$

Question 9 ***Prove that :*** $\sin^{-1}\frac{3}{5} - \sin^{-1}\frac{8}{17} = \cos^{-1}\frac{84}{85}$

Question 10 ***Prove that*** $\tan^{-1}\left(\frac{x}{\sqrt{a^2-x^2}}\right) = \sin^{-1}\left(\frac{x}{a}\right), \; |x| < a$

Question 11 **Prove that :** $\tan\frac{1}{2}\left[\sin^{-1}\frac{2x}{1+x^2}+\cos^{-1}\frac{1-y^2}{1+y^2}\right]$

$=\frac{x+y}{1-xy}$, **if |x| < 1, y > 0 and xy > 1**

Question 12 **Prove that :** $2\tan^{-1}\frac{1}{2}+\tan^{-1}\frac{1}{7}=\tan^{-1}\frac{31}{17}$

Question 13 **Prove that :** $\sin^{-1}\frac{3}{5}-\sin^{-1}\frac{8}{17}=\cos^{-1}\frac{84}{85}$

Question 14 **Prove that** $2\sin^{-1}x=\sin^{-1}\left(2x\sqrt{1-x^2}\right),\quad |x|\le\frac{1}{\sqrt{2}}$

Question 14 **Prove that** $\tan^{-1}\left(\frac{\sqrt{1+\cos x}+\sqrt{1-\cos x}}{\sqrt{1+\cos x}-\sqrt{1-\cos x}}\right)=\frac{\pi}{4}+\frac{x}{2}$, where, $0<x<\frac{\pi}{2}$

Question 15 **Prove that :**

$$\sin[\cot^{-1}\{\cos(\tan^{-1}x)\}]=\sqrt{\frac{x^2+1}{x^2+2}}$$

CHAPTER 3 & 4

MATRICES AND DETERMINANTS

(1 Mark Questions)

1. Let A be a square matrix of order 3×3 than $|KA|$ is equal to :

   (A) $k|A|$ (B) $K^2|A|$ (C) $k^3|A|$ (D) $3k|A|$

2. If a, b, c are in A.P. then determinant

   $$\begin{vmatrix} x+2 & x+3 & x+2a \\ x+3 & x+4 & x+2b \\ x+4 & x+5 & x+2c \end{vmatrix} \text{ is}$$

   (A) 0 (B) 1 (C) x (D) $2x$

3. T_p, T_q, T_r are the p[th], q[th] and r[th] terms of an A.P. then $\begin{vmatrix} T_p & T_q & T_r \\ p & q & r \\ 1 & 1 & 1 \end{vmatrix}$ equals

   (A) 1 (B) -1 (C) 0 (D) p+q+r

4. The value of $\begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{vmatrix}$, ω being a cube root of unity is

   (A) 0 (B) 1 (C) ω^2 (D) ω

5. If a+b+c=0, one root of

   $$\begin{vmatrix} a-x & c & b \\ c & b-x & a \\ b & a & c-x \end{vmatrix} = 0$$

   (A) x=1 (B) x=2 (C) $x = a^2+b^2+c^2$ (D) x=0

6. The roots of the equation

   $$\begin{vmatrix} a-x & b & c \\ 0 & b-x & 0 \\ 0 & b & c-x \end{vmatrix} = 0 \text{ are}$$

   (A) a and b (B) b and c (C) a and c (D) a,b and c

7. Value of $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$ is

(A) (a-b) (b-c) (c-a) (B) $(a^2-b^2)(b^2-c^2)(c^2-a^2)$

(C) (a-b+c) (b-c+a) (c-a+b) (D) None of these

8. If A and B are any 2 x 2 matrices, then det (A+B)=0 implies

(A) det A + det B=0 (B) det A=0 or det B=0

(C) det A=0 and det B=0 (D) None of these

9. If A and B are 3 x 3 matrices then AB=0 implies

(A) A=0 and B=0 (B) |A|=0 and |B|=0

(C) Either |A|=0 or |B|=0 (D) A=0 or B=0

10. The value of λ for which the system of equations :-

$x + y + 2 = 6,\ x + 2y + 3z = 10,\ x + 4y + \lambda z = 1$ has a unique solution is

(A) $\lambda \neq -7$ (B) $\lambda \neq 7$

(C) $\lambda = 7$ (D) $\lambda = -7$

11. If the system of the equation :

$x - ky - z = 0, kx - y - z = 0, x + y - z = 0$ has a non-zero solution, then the possible values of *k* are :

(A) -1, 2 (B) 1,2

(C) 0, 1 (D) -1, 1

12. If A is a 3 x 3 non singular matrix than det [adj. (A)] is equal to

(A) $(\det A)^2$ (B) $(\det A)^3$

(C) $\det A$ (D) $(\det A)^{-1}$

13. If A is an invertible matrix of order *n,* then the determinant of Adj. A =

(A) $|A|^n$ (B) $|A|^{n+1}$

(C) $|A|^{n-1}$ (D) $|A|^{n+2}$

14. The value of $\begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix}$ is

(A) a+b+c (B) 1

(C) 0 (D) abc

15. If A^2-A+I=0 then the inverse of A is

(A) A (B) A+I

(C) I-A (D) A-I

(2 Mark Questions)

1. Find a matrix X such that

$$3A - 2B + X = 0 \text{ where}$$

$$A = \begin{bmatrix} 4 & 2 \\ 1 & 3 \end{bmatrix}, \quad B = \begin{bmatrix} -2 & 1 \\ 3 & 2 \end{bmatrix}$$

2. Find x and y, if

$$2\begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$$

3. Solve $[2x \;\; 3] \begin{bmatrix} 1 & 2 \\ -3 & 0 \end{bmatrix} \begin{bmatrix} x \\ 3 \end{bmatrix} = 0$

4. Show that $A+A^T$ is symmetric Matrix

$A = \begin{bmatrix} 3 & -4 \\ 7 & 8 \end{bmatrix}$. Where A^T is the transpose of A.

5. If $A = \begin{bmatrix} \sin\alpha & \cos\alpha \\ -\cos\alpha & \sin\alpha \end{bmatrix}$, then prove that A'A=I

6. Show that the matrix A is skew – symmetric, where

$$A=\begin{bmatrix}0 & a & b\\ -a & 0 & c\\ -b & -c & 0\end{bmatrix}$$

7. Find the value of k if area of the triangle is 4 square units and vertices are (-2, 0) (0,4), (0, k)

8. If $A=\begin{bmatrix}1 & 0 & 1\\ 0 & 1 & 2\\ 0 & 0 & 4\end{bmatrix}$, then show that |3A|=27|A|

9. Without actual expansion, Prove that the determinant A Vanish.

Where $|A|=\begin{vmatrix}1 & a & b+c\\ 1 & b & c+a\\ 1 & c & a+b\end{vmatrix}$

10. If $A=\begin{bmatrix}1 & 2 & 3\\ 2 & 3 & 2\\ 3 & 3 & 4\end{bmatrix}$

Find (adj. A)

11. Find the inverse of matrix $\begin{bmatrix}4 & -2\\ 3 & 1\end{bmatrix}$

12. If $A=\begin{bmatrix}2 & -3\\ 3 & 4\end{bmatrix}$, show that $A^2-6A+17I_2=0$

(4 Mark Questions)

1. Construct a 3 × 2 matrix A = $[a_{ij}]$ whose elements are given by $a_{ij}=\begin{cases} i+j & \text{if } i=j\\ \dfrac{|i-2j|}{2} & \text{if } i<j\end{cases}$

2. If $A = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$ and B = [-2 -1 -4], very that (AB)' = B' A'

3. Express the matrix $\begin{bmatrix} 3 & 3 & 1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix} = P + Q$ where P is a symmetric and Q is a skew-symmetric matrix.

4. If $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ verify prove that $A^n = \begin{bmatrix} \cos n\theta & \sin n\theta \\ -\sin n\theta & \cos n\theta \end{bmatrix}$ where n is a natural number.

5. Let $A = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$ $B = \begin{bmatrix} 5 & 2 \\ 7 & 4 \end{bmatrix}$ $C = \begin{bmatrix} 2 & 5 \\ 3 & 8 \end{bmatrix}$ find a matrix D such that CD-AB=O

6 Find the value of x such that $\begin{bmatrix} 1 & x & 1 \end{bmatrix} \begin{bmatrix} 1 & 3 & 2 \\ 2 & 5 & 1 \\ 15 & 3 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ x \end{bmatrix} = 0$

7 Prove that the product of the matrices $\begin{bmatrix} \cos^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta \end{bmatrix}$ *and* $\begin{bmatrix} \cos^2\phi & \cos\phi\sin\phi \\ \cos\phi\sin\phi & \sin^2\phi \end{bmatrix}$ the null matrix, when θ and ϕ differ by an odd multiple of $\frac{\pi}{2}$.

8. If $A = \begin{bmatrix} 5 & 3 \\ 12 & 7 \end{bmatrix}$ show that $A^2 - 12A - I = 0$. Hence find A^{-1}.

9. If $A = \begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix}$, find x and y such that $A^2 - xA + yI = 0$.

10. If $A = \begin{bmatrix} 2 & 3 \\ 1 & -4 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$ then show that $(AB)^{-1} = B^{-1}A^{-1}$.

11. Test the consistency of the following system of equations by matrix method :

$3x - y = 5;\ 6x - 2y = 3$

12. Using elementary row transformations, find the inverse of the matrix $A = \begin{bmatrix} 6 & -3 \\ -2 & 1 \end{bmatrix}$, if possible.

13. By using elementary column transformation, find the inverse of $A = \begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix}$.

14. If $A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$ and $A + A' = I$, then find the general value of α

Using properties of determinants, prove the following : Q. 15 to Q 21

15. $\begin{vmatrix} x+2 & x+3 & x+2a \\ x+3 & x+4 & x+2b \\ x+4 & x+5 & x+2c \end{vmatrix} = 0$ if a, b, c are in A.P.

16. $\begin{vmatrix} \sin\alpha & \cos\alpha & \sin(\alpha+\delta) \\ \sin\beta & \cos\beta & \sin(\beta+\delta) \\ \sin\gamma & \cos\gamma & \sin(\gamma+\delta) \end{vmatrix} = 0$

17. $\begin{vmatrix} a^2 & bc & c^2+ac \\ a^2+ab & b^2 & ac \\ ab & b^2+bc & c^2 \end{vmatrix} = 4a^2b^2c^2$

18. $\begin{vmatrix} x+a & b & c \\ a & x+b & c \\ a & b & x+c \end{vmatrix} = x^2(a+b+c+d)$

19. Show that :

$$\begin{vmatrix} 1 & 1 & 1 \\ x^2 & y^2 & z^2 \\ yz & zx & xy \end{vmatrix} = (y-z)(z-x)(x-y)(yz+zx+xy).$$

20. (i) If the points (a, b) (a´, b´) and (a – a´, b – b´) are collinear. Show that ab´ = a´b.

(ii) If $A = \begin{bmatrix} 2 & 5 \\ 2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & -3 \\ 2 & 5 \end{bmatrix}$ verify that $|AB| = |A||B|$

21 Solve the following equation for x.

$$\begin{vmatrix} a+x & a-x & a-x \\ a-x & a+x & a-x \\ a-x & a-x & a+x \end{vmatrix} = 0$$

LONG ANSWER TYPE QUESTIONS (6 MARKS)

1. Obtain the inverse of the following matrix using elementary row operations $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$

2. Using matrix method, solve the following system of linear equations :

   x – y + 2z = 1, 2y – 3z = 1, 3x – 2y + 4z = 2.

3. Solve the following system of equations by matrix method, where x ≠ 0, y ≠ 0, z ≠ 0

   $\frac{2}{x} - \frac{3}{y} + \frac{3}{z} = 10$,, $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 10$, $\frac{3}{x} - \frac{1}{y} + \frac{2}{z} = 13$. where $x \neq 0,\ y \neq 0,\ z \neq 0$

4. Find A^{-1}, where $A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix}$, hence solve the system of linear equations :

   x + 2y – 3z = – 4

   2x + 3y + 2z = 2

   3x – 3y – 4z = 11

5. The sum of three numbers is 2. If we subtract the second number from twice the first number, we get 3. By adding double the second number and the third number we get 0. Represent it algebraically and find the numbers using matrix method.

6. Compute the inverse of the matrix.

$$A = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 5 \end{bmatrix} \text{ and verify that } A^{-1} A = I_3$$

7. If the maxtric $A = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}$ and $B^{-1} = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 3 & -1 \\ -1 & 0 & 2 \end{bmatrix}$, then compute $(AB)^{-1}$

8. Using matrix method, solve the following system of linear equations :

$$2x - y = 4,\ 2y + z = 5,\ z + 2x = 7$$

9 Find the inverse of the matrix A $\begin{vmatrix} 1 & 3 & 0 \\ 0 & 2 & 1 \end{vmatrix}$ by using elementary column transformations.

10. Let $A = \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$ and $f(x) = x^2 - 4x + 7$. Show that $f(A) = 0$. Use this result to find A^5

11. If $A = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$, verify that $A . (\text{adj } A) = (\text{adj } A) . A = |A|\, I_3$.

12. For the matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$, verify that $A^3 - 6A^2 + 9A - 4I = 0$, hence find A^{-1}.

13. By using properties of determinants prove the following :

$$\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = (1+a^2+b^2)^3 .$$

14. $$\begin{vmatrix} (y+z)^2 & xy & zx \\ xy & (x+z)^2 & yz \\ xz & yz & (x+y)^2 \end{vmatrix} = 2xyz(x+y+z)^{2.}$$

15. $$\begin{vmatrix} a & a+b & a+b+c \\ 2a & 3a+2b & 4a+3b+2c \\ 3a & 6a+3b & 10a+6b+3c \end{vmatrix} = a^3 .$$

16. If x, y, z are different and $\begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix}$ is equal to zero. Show that $xyz = -1$.

CHAPTER-5
CONTINUITY AND DIFFERENTIATION

(1 Mark Questions)

1. If $f(x) = |x|$ then $f(x)$ is

   (A) Continuous for all *x* (B) Continuous only at certain points

   (C) Differentiable at all points (D) None of these

2. Find k, $f(x) = \begin{cases} kx^2, \text{ if } & x \le 2 \\ 3, \text{ if } & x > 2 \end{cases}$ is continuous at x=2

   (A) 1/2 (B) 3/4

   (C) 6 (D) 1

3. $\underset{h \to 0}{Lt} \dfrac{\cos^2(x+h) - \cos^2 x}{h}$ is equal to

   (A) $\cos^2 x$ (B) $-\sin 2x$

   (C) $\sin x \cos x$ (D) $2 \sin x$

4. If $f(x) = \begin{cases} \dfrac{1 - \sqrt{2} \sin x}{\pi - 4x} & if \quad x \ne \dfrac{\pi}{4} \\ a & if \quad x = \dfrac{\pi}{4} \end{cases}$

   is continuous at $x = \dfrac{\pi}{4}$ then, 'a' equals

   (A) 4 (B) 2

   (C) 1 (D) $\dfrac{1}{4}$

5. The derivative of log $\left(e^{x^2}\right)$ is

   (A) $\dfrac{1}{e^{x^2}}$ (B) e^{x^2} (C) $2x$ (D) None of these

6. If $\sqrt{x}+\sqrt{y}=\sqrt{a}$ then $\frac{dy}{dx}$ equals

(A) $-\sqrt{\frac{y}{x}}$ (B) $-\sqrt{\frac{x}{y}}$

(C) $\sqrt{\frac{y}{x}}$ (D) $\sqrt{\frac{x}{y}}$

7. The derivative of $f(x) = \tan^{-1}\left(\frac{x}{2}\right)$

(A) $\frac{4}{4+x^2}$ (B) $\frac{2}{4+x^2}$ (C) $\frac{1}{2+x^2}$ (D) $\frac{2}{\sqrt{4-x^2}}$

8. If $y=\sqrt{\sin + \sqrt{\sin x+\sqrt{\sin x+..to\infty}}}$ then, $\frac{dy}{dx}$ is equal to

(A) $\frac{\sin x}{y}$ (B) $\frac{\sin x+\cos x}{2y+1}$

(C) $\frac{\cos x}{2y-1}$ (D) $\frac{\cos x}{1-2y}$

9. If $x^2+y^2=1$, then

(A) $yy''-2y^2-1=0$ (B) $yy''+y'^2+1=0$

(C) $yy''-y'^2-1=0$ (D) $yy''-2y'^2+1=0$

10. The derivative of $\left(\tan^{1} x+\sec^{-1} x\right)$

(A) $\frac{1}{8}$ (B) $\frac{1}{4}$

(C) $\frac{1}{2}$ (D) 0

11. Find K, $\begin{cases} 2x+1 & x<2 \\ k & x=2 \\ 3x-1 & x>2 \end{cases}$ to be continuous at x=2 is

(A) 3 (B) -5 (C) 0 (D) 5

12. If $y=x^y$,then $\frac{dy}{dx}$ equals

(A) $\frac{x^2}{y(1-y\log x)}$ (B) $\frac{y^2}{x(1-x\log y)}$

(C) $\frac{y^2}{x(1-x\log x)}$ (D) $\frac{y^2}{x(1-y\log x)}$

13. If x=at^2, y=2at, then $\frac{dy}{dx}$ equals

(A) $\frac{1}{t}$, (B) 2at (C) 2a (D) t

14. The derivative of x^6 w.r.t. x^3 is

(A) $6x^6$ (B) $3x^2$ (C) $2x^3$ (D) x^2

15. If $f(x)=10^x$, then $f'(x)$equals:

(A) 10^x (B) $x^{10}\log_{10}$ (C) $\frac{10^x}{\log 10}$ (D) $10^x \log 10$

16. If $f(x)=\begin{cases}3x-9, & 0\le x\le 2\\ 2x+\lambda & 2<x\le 3\end{cases}$ is continuous at x=2, then what is the value of λ ?

(A) 1 (B) -1 (C) 2 (D) -2

17. If $f(x)=\frac{1}{3x-1}$, then for x=0

(A) $f'(x)=0$ (B) $f'(x)<0$ (C) $f'(x)>0$ (D) $f'(x)=f(x)$

18. Derivative of $\tan^{-1}(\cot x)$ w.r.t x equals

(A) -1 (B) 1 (C) $\tan x$ (D) $\cot x$

19. The value of k for the function :

$$f(x)=\begin{cases}\frac{x^2-4}{x-2}, & x\ne 2\\ k, & x=2\end{cases}$$

to be continuous is

(A) 0 (B) 2 (C) 3 (D) 4

20. The derivative of $\sqrt{\log(\sin x)}$ is:

(A) $\frac{\cot x}{2\sqrt{\log(\sin x)}}$ (B) $\cot x$ (C) $\frac{1}{\sqrt{\log(\sin x)}}$ (D) $\tan x$

(2 Mark Questions)

1. For what value of 'p' is the following function continuous at x=0 :

$$f(x)=\begin{cases} \frac{1-\cos 4x}{8x^2} & x\neq 0 \\ p & x=0 \end{cases}$$

2. Discuss the continuity of the function

$$f(x)=\begin{cases} \frac{|x-2|}{2-x} & x\neq 2 \\ -1 & x=2 \end{cases} \quad \text{at } x=2$$

3. Differentiate $\tan^{-1}\sqrt{\frac{1+\sin x}{1-\sin x}}$ w.r.t. x

4. Find $\frac{dy}{dx^1}$ if $xy=e^{x-y}$

5. Find $\frac{dy}{dx^1}$ if $x=a\cos\theta,\ y=a\sin\theta$

6. Differentiate x^3 w.r.t x^3.

7. Differentiate

$$y=\sqrt{\log x+\sqrt{\log x+\sqrt{\log x+.........}}} \quad \text{w.r.t. } x$$

8. Differentiate $x^{\sin x^{-1}x}$ w.r.t x

9. Differentiate $\sin^{-1}\left(\frac{2x}{1+x^2}\right), -1<x<1$ w.r.t. x

10. Find $\frac{dy}{dx}$, if $\frac{x^2}{a^2}+\frac{y^2}{b^2}=2018$

11. Examine the derivability of :

$$f(x):\begin{cases} x^2 \sin\frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

12. Locate the point of discontinuity of the following function

$$f(x)=\begin{bmatrix} x^3 - x^2 + 2x - 2 & if \quad x \neq 1 \\ 4 & if \quad x = 1 \end{bmatrix}$$

(4 Mark Questions)

Discuss the continuity of following functions at the indicated points.

(1) $f(x)=\begin{cases} x\sin\frac{1}{x}, & x\neq 0 \\ 0 & x=0 \end{cases}$ **at x=0**

(2) $g(x) = \begin{cases} \frac{\sin 2x}{3x}, & x \neq 0 \\ \frac{3}{2} & x = 0 \end{cases}$ at $x = 0$.

(3) $f(x) = \begin{cases} x^2 \cos(1/x) & x \neq 0 \\ 0 & x = 0 \end{cases}$ at $x = 0$.

(4) $f(x) = |x| + |x - 1|$ at $x = 1$.

(5) $f(x) = \begin{cases} x - [x], & x \neq 1 \\ 0 & x = 1 \end{cases}$ at $x = 1$.

6. For what value of K, $f(x) = \begin{bmatrix} 3x^2 - kx + 5, & 0 \leq x < 2 \\ 1 - 3x & 2 \leq x \leq 3 \end{bmatrix}$ is continuous $\forall x \in [0, 3]$.

7. If the function $f(x)$ given by

$$f(x)=\begin{cases} 2ax+b & \text{if } x>1 \\ 5 & \text{if } x=1 \\ 3ax-b & \text{if } x<1 \end{cases}$$

is continuous at $x = 1$, Find the value of *a* and *b*

8. Prove that f(x) = |x + 1| is continuous at x = –1, but not derivable at x = –1.

9. For what value of p,

$$f(x)=\begin{cases} x^p \sin(1/x), & x \neq 0 \\ 0 & x = 0 \end{cases}$$ is derivable at x = 0.

10 **Discuss the continuity of the function**

$$f(x)=\begin{cases} \dfrac{1-\cos 4x}{x^2} & x \neq 0 \\ 2 & x = 0 \end{cases} \quad \textbf{at} \quad x=0$$

11 Find $\frac{dy}{dx}$ if $y = 1 + 2\left(\frac{x}{x+1}\right)^2 + 3\left(\frac{x}{x+1}\right)^3$

12 Find the derivative of $\tan^{-1}\left(\frac{2x}{1-x^2}\right)$ w.r.t. $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$.

13. Find the derivative of $\log_e(\sin x)$ w.r.t. $\log_e(\cos x)$.

14. **Differentiate** $y = \cot^{-1}\left(\frac{x}{\sqrt{1+x^2}-1}\right)$ **with respect to** x

15. If $y = \tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{.-x}}{\sqrt{1+x}+\sqrt{1-x}}\right)$, find $\frac{dy}{dx}$.

16. If $x = ae^t$ (sint – cos t)

$y = ae^t$ (sint + cost) then show that $\frac{dy}{dx}$ at $x = \frac{\pi}{4}$ is 1 .

17 **If** $y = \tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right)$, find $\frac{dy}{dx}$

18 **Find $\frac{dy}{dx}$ if** $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$

19 **If x =** $\frac{\sin^3 t}{\sqrt{\cos 2t}}$ **, y =** $\frac{\cos^3 t}{\sqrt{\cos 2t}}$ **, find** $\frac{dy}{dx}$

20 **Find the derivative of** $\sqrt{e^{\sqrt{x}}}$

21 Give an example of the function which is continuous everywhere but not differentiable at exactly two points.

22. Discuss the applicability of Rolle's theorem for the following function on the indicated interval : f (x) = |x| on [–1, 1]

23 Discuss the applicability of Rolle's theorem for the following function on the indicated interval :

$f(x) = 3 + (x - 2)^{2/3}$ on [1, 3]

24 It is given that for the function f given by

f(x) = $x^3 + bx^2 + ax + 1,\ x \in [1,3]$

Rolle's theorem holds with c = $2 + \frac{1}{\sqrt{3}}$. Find the values of a and b.

25 Verify the mean value theorem for the function $f(x) = e^{x-1}$, in[1,2]

26 Verify the mean value theorem for the function $f(x) = \sqrt{x^2 - 4}$ for the interval [2,5]

27 Using Lagrange's mean value theorem, prove that

$\frac{b-a}{b} < \log\left(\frac{b}{a}\right) < \frac{b-a}{a}$, where 0 < a < b.

CHAPTER-6

APPLICATION OF DERIVATIVES

(3 Mark Questions)

1. A particle cover along the curve $6y = x^3 + 2$. Find the points on the curve at which the y co-ordinate is changing 8 times as fast as the x co-ordinate.

2. A ladder 5 metres long is leaning against a wall. The bottom of the ladder is pulled along the ground away from the wall as the rate of 2 cm/sec. How fast is its height on the wall decreasing when the foot of the ladder is 4 metres away from the wall?

3. A balloon which always remain spherical is being inflated by pumping in 900 cubic cm of a gas per second. Find the rate at which the radius of the balloon increases when the radius is 15 cm.

4. A man 2 meters high walks at a uniform speed of 5 km/hr away from a lamp post 6 metres high. Find the rate at which the length of his shadow increases.

5. Water is running out of a conical funnel at the rate of 5 cm^3/sec. If the radius of the base of the funnel is 10 cm and attitude is 20 cm. Find the rate at which the water level is dropping when it is 5 cm from the top.

6. The length x of a rectangle is decreasing at the rate of 5 cm/sec and the width y is increasing as the rate of 4 cm/sec when $x = 8$ cm and $y = 6$ cm. Find the rate of change of

   (a) Perimeter (b) Area of the rectangle.

7. Sand is pouring from a pipe as the rate of $12cm^2$ /sec. The falling sand forms a cone on the ground in such a way that the height of the cone is always one-sixth of the radius of the base. How fast is the height of the sand cone is increasing when height is 4 cm?

8. The area of an expanding rectangle is increasing at the rate of 48 cm^2/sec. The length of the rectangle is always equal to the square of the breadth. At what rate lies the length increasing at the instant when the breadth is 4.5 cm?

9. Find a point on the curve $y = (x - 3)^2$ where the tangent is parallel to the line joining the points (4, 1) and (3, 0).

12. Find the equation of the normal at the point (am^2, am^3) for the curve $ay^2 = x^3$.

13. Show that the curves $4x = y^2$ and $4xy = k$ cut as right angles if $k^2 = 512$.

14. Find the equation of the tangent to the curve $y = \sqrt{3x - 2}$ which is parallel to the line $4x - y + 5 = 0$.

16. Find the points on the curve $4y = x^3$ where slope of the tangent is $\frac{16}{3}$.

17. Show that $\frac{x}{a}+\frac{y}{b}=1$ touches the curve $y = be^{-x/a}$ at the point where the curve crosses the y-axis.

18. Find the equation of the tangent to the curve given by $x = a \sin^3 t$, $y = b \cos^3 t$ at a point where $t = \frac{\pi}{2}$.

19. Find the intervals in which the function $f(x) = \log(1 + x) - \frac{x}{1+x}$, $x > -1$ is increasing or decreasing.

20. Find the intervals is which the function $f(x) = x^3 - 12x^2 + 36x + 17$ is

    (a) increasing (b) decreasing.

21. Prove that the function $f(x) = x^2 - x + 1$ is neither increasing nor decreasing in [0, 1].

22. Find the intervals on which the function $f(x)=\frac{x}{x^2+1}$ is decreasing.

23. Prove that the functions given by $f(x) = \log \cos x$ is strictly decreasing on $\left(0, \frac{\pi}{2}\right)$ and strictly increasing on $\left(\frac{\pi}{2}, \pi\right)$.

24. Find the intervals on which the function $f(x)=\frac{\log x}{x}$ increasing or decreasing.

25. Find the intervals in which the function $f(x) = \sin^4 x + \cos^4 x,\ 0 \leq x \leq \frac{\pi}{2}$ is increasing or decreasing.

26. Find the least value of 'a' such that the function $f(x) = x^2 + ax + 1$ is strictly increasing on (1, 2).

27. Find the interval in which the function $f(x) = 5x^{\frac{3}{2}} - 3x^{\frac{5}{2}},\ x > 0$ is strictly decreasing.

Using differentials, find the approximate value of (Q. No. 28 to 30).

28. $(255)^{\frac{1}{4}}$.

29. $(66)^{\frac{1}{3}}$.

30. $\sqrt{25.3}$

(6 Mark Questions)

1. Show that of all rectangles inscribed in a given fixed circle, the square has the maximum area.
2. Find two positive numbers x and y such that their sum is 35 and the product x^2y^5 is maximum.
3. Show that of all the rectangles of given area, the square has the smallest perimeter.
4. Show that the right circular cone of least curved surface area and given volume has an altitude equal to $\sqrt{2}$ times the radium of the base.
5. Show that the semi vertical angle of right circular cone of given surface area and maximum volume is $\sin^{-1}\left(\frac{1}{3}\right)$.
6. Show that the right triangle of maximum area that can be inscribed in a circle is an isosceles triangle.

7. Prove that the volume of the largest cone that can be inscribed in a sphere of radius R is $\frac{8}{27}$ of the volume of the sphere.

8. Find the interval in which the function f given by $f(x) = \sin x + \cos x, 0 \le x \le 2\pi$ is strictly increasing or strictly decreasing.

9. Find the intervals in which the function $f(x) = (x + 1)^3 (x - 3)^3$ is strictly increasing or strictly decreasing.

10. Find the local maximum and local minimum of $f(x) = \sin 2x - x, -\frac{\pi}{2} < x < \frac{\pi}{2}$.

11. Find the intervals in which the function $f(x) = 2x^3 - 15x^2 + 36x + 1$ is strictly increasing or decreasing. Also find the points on which the tangents are parallel to x-axis.

12. A solid is formed by a cylinder of radius r and height h together with two hemisphere of radius r attached at each end. It the volume of the solid is constant but radius r is increasing at the rate of $\frac{1}{2\pi}$ metre /min. How fast must h (height) be changing when r and h are 10 metres.

13. Find the equation of the normal to the curve

$x = a (\cos \theta + \theta \sin \theta) ; y = a (\sin \theta - \theta \cos \theta)$ at the point θ and show that its distance from the origin is a.

14. For the curve $y = 4x^3 - 2x^5$, find all the points at which the tangent passes through the origin.

15. Find the equation of the normal to the curve $x^2 = 4y$ which passes through the point (1, 2).

16. Find the equation of the tangents at the points where the curve $2y = 3x^2 - 2x - 8$ cuts the x-axis and show that they make supplementary angles with the x-axis.

18. A window is in the form of a rectangle surmounted by an equilateral triangle. Given that the perimeter is 16 metres. Find the width of the window in order that the maximum amount of light may be admitted.

19. A square piece of tin of side 18 cm is to be made into a box without top by cutting a square from each cover and folding up the flaps to form the box. What should be the side of the square to be cut off so that the value of the box is the maximum points.

20. A window is in the form of a rectangle is surmounted by a semi circular opening. The total perimeter of the window is 30 metres. Find the dimensions of the rectangular part of the window to admit maximum light through the whole opening.

21. An open box with square base is to be made out of a given iron sheet of area 27 sq. meter show that the maximum value of the box is 13.5 cubic metres.

22. A wire of length 28 cm is to be cut into two pieces. One of the two pieces is to be made into a square and other in to a circle. What should be the length of two pieces so that the combined area of the square and the circle is minimum?

23. Show that the height of the cylinder of maximum volume which can be inscribed in a sphere of radius R is $\frac{2R}{\sqrt{3}}$. Also find the maximum volume.

24. Show that the altitude of the right circular cone of maximum volume that can be inscribed is a sphere of radius r is $\frac{4r}{3}$.

25. Prove that the surface area of solid cuboid of a square base and given volume is minimum, when it is a cube.

26. Show that the volume of the greatest cylinder which can be inscribed in a right circular cone of height h and semi-vertical angle α is $\frac{4}{27}\pi h^3 \tan^2 \alpha$.

CHAPTER-7

INTEGRALS

(1 Mark Questions)

Evaluate the following integrals

1. If $\frac{d}{dx}(f(x)) = 4x^3 - \frac{3}{x^4}$ such that $f(2) = 0$, then $f(x)$ is

(A) $x^4 + \frac{1}{x^3} - \frac{129}{8}$ (B) $x^3 + \frac{1}{x^4} + \frac{129}{8}$

(C) $x^4 + \frac{1}{x^3} + \frac{129}{8}$ (D) $x^3 + \frac{1}{x^4} - \frac{129}{8}$

2. $\int \frac{1}{1+e^{-x}} dx$ is equal to :

(A) $\log(1+e^{-x}) + c$ (B) $\log(1+e^{x}) + c$

(C) $x - e^{-x} + c$ (D) None of these

3. $\int \left(\sin^{-1}\sqrt{x} + \cos^{-1}\sqrt{x}\right)dx$ is equal to

(A) $\frac{\pi}{2}x + c$ (B) $\frac{\pi}{4}x + c$

(C) $x + c$ (D) $\pi x + c$

4. $\int \left(\frac{10x^{a} + 10^{x} \log_e 10}{x^{10} + 10^{x}} \right) dx$ equals

(A) $10^{x} - x^{10} + c$ (B) $10^{x} + x^{10} + c$

(C) $\left(10^{x} - x^{10}\right)^{-1} + c$ (D) $\log\left(10^{x} + 10^{x}\right) + c$

5. $\int \frac{1}{\sin^2 x \cos^2 x} dx$ equals

(A) $\tan x + \cot x + c$ (B) $\tan x - \cot x + c$

(C) $\tan x \cot x + c$ (D) $\tan x - \cot 2x + c$

6. $\int \frac{1}{x^2 + 2x + 2} dx$ equals

(A) $x \tan^{-1}(x+1) + c$ (B) $\tan^{-1}(x+1) + c$

(C) $(x+1)\tan^{-1} x + c$ (D) $\tan^{-1} x + c$

7. $\int \frac{1}{\sqrt{9x - 4x^2}} dx$ equals

(A) $\frac{1}{9}\sin^{-1}\left(\frac{9x-8}{8}\right) + c$ (B) $\frac{1}{2}\sin^{-1}\left(\frac{8x-9}{8}\right) + c$

(C) $\frac{1}{3}\sin^{-1}\left(\frac{9x-8}{8}\right) + c$ (D) $\frac{1}{2}\sin^{-1}\left(\frac{9x-8}{9}\right) + c$

8. $\int \frac{x(1+x)}{\cos^2\left(xe^{x}\right)} dx$ equals

(A) $-\cot\left(xe^{x}\right) + c$ (B) $\tan\left(xe^{x}\right) + c$

(C) $\tan\left(e^{x}\right) + c$ (D) $\cot\left(e^{x}\right) + c$

9. $\int \frac{x\,dx}{(x-1)(x-2)}$ equals

(A) $\log\left|\frac{(x-1)^2}{x-2}\right| + c$ (B) $\log\left|\frac{(x-2)^2}{x-1}\right| + c$

(C) $\log\left|\left(\frac{x-1}{x-2}\right)^2\right| + c$ (D) $\log\left|(x-1)(x-2)\right| + c$

10. $\int \frac{1}{x\left(x^2+1\right)} dx$ equals

(A) $\log|x| - \frac{1}{2}\log\left(x^2+1\right) + c$ (B) $\log|x| + \frac{1}{2}\log\left(x^2+1\right) + c$

(C) $-\log|x|+\frac{1}{2}\log(x^2+1)+c$ (D) $\frac{1}{2}\log|x|+\log(x^2+1)+c$

11. $\int x^2 e^{x^3} dx$ equals

(A) $\frac{1}{3}e^{x^3}+c$ (B) $\frac{1}{3}e^{x^2}+c$

(C) $\frac{1}{2}e^{x^3}+c$ (D) $\frac{1}{2}e^{x^2}+c$

12. $\int e^x \sec x(1+\tan x)dx$ equals

(A) $e^x \cos x + c$ (B) $e^x \sec x + c$

(C) $e^x \sin x + c$ (D) $e^x + \tan x + c$

13. $\int \sqrt{1+x^2}\,dx$ equals

(A) $\frac{x}{2}\sqrt{1+x^2}+\frac{1}{2}\log\left|x+\sqrt{1+x^2}\right|+c$

(B) $\frac{2}{3}(1+x^2)^{3/2}+c$

(C) $\frac{2}{3}x(1+x^2)^{3/2}+c$

(D) $\frac{x^2}{2}\sqrt{1+x^2}+\frac{1}{2}x^2\log\left|x+\sqrt{1+x^2}\right|+c$

14. If $\int \frac{5^{\frac{1}{x}}}{x^2}dx = k\,5^{\frac{1}{x}}$ then the value of *k* is :

(A) log 5 (B) - log 5

(C) $-\frac{1}{\log 5}$ (D) $\frac{1}{\log 5}$

15. $\int_{-2}^{2}|x|dx=$

(A) 0 (B) 1 (C) 2 (D) 4

16. $\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}}\sin^5 x\,dx$ has value

(A) 0 (B) -1 (C) 1 (D) None of these

17. If $\int_0^a \frac{dx}{1+4x^2} = \frac{\pi}{8}$ then a is equal to

(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{4}$ (C) 1 (D) $\frac{1}{2}$

18. $\int_0^{\frac{\pi}{2}} \frac{(\sin x)^{2018}}{(\sin x)^{2018} + (\cos x)^{2018}} dx$ equals

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{2}$ (C) $2018(\sin x)^{2019}$ (D) None of these.

19. $\int_3^6 \frac{\sqrt{x}}{\sqrt{9-x}+\sqrt{x}}\, dx$ is equal to

(A) $\frac{1}{2}$ (B) $\frac{3}{2}$ (C) 2 (D) 1

20. The value of $I = \int_0^{\frac{\pi}{2}} \frac{(\sin x + \cos x)^2}{\sqrt{1+\sin 2x}}\, dx$ is

(A) 0 (B) 1 (C) 2 (D) 3

(2 Mark Questions)

1. Evaluate : $\int \frac{\sin 2x\, dx}{a\cos^2 x + b\sin^2 x}$

2. Evaluate : $\int \frac{(x+1)e^x}{\sin^2(xe^x)} dx$

3. Integrate $\int \sin^3 x\, dx$

4. Evaluate $\int \frac{dx}{e^x + e^{-x}}$

5. Evaluate $\int \sqrt{\frac{a+x}{a-x}}\, dx$

6. Integrate $\int e^x \sin x\, dx$

7. Integrate $\int e^x \left(\tan^{-1} x + \frac{1}{1+x^2} \right) dx$

8. Evaluate $\int_0^1 |3x-1| dx$

9. Evaluate $\int_0^{\pi/2} \frac{\sin^{2018} x}{\sin^{2018} x + \cos^{2018} x} dx$

10. Evaluate $\int_{-8}^{-8} \left(\sin^{93} x + x^{295} \right) dx$

11. Evaluate $\int_0^1 x(1-x)^n dx$

12. Evaluate $\int_{-\pi/2}^{\pi/2} \sin^5 x \, dx$

13. Evaluate $\int \frac{dx}{x(x^7+1)}$

14. Evaluate $\int \frac{1}{x^2-16} dx$

15. Evaluate $\int \frac{1}{(x+1)(x+2)} dx$

16. Evaluate $\int x \, e^{2x} \, dx$

17. Evaluate $\int \log x \, dx$

18. Evaluate $\int \sin 3x \cos 4x \, dx$

19. Evaluate $\int \frac{dx}{x^{1/2} + x^{1/3}}$

20. Evaluate $\int \frac{\sin(x-a) \, dx}{\sin x}$

(4 Mark Questions)

1. (i) $\int \frac{x \operatorname{cosec}\left(\tan^{-1} x^2\right)}{1 + x^4} dx.$ (ii) $\int \frac{\sqrt{x+1} - \sqrt{x-1}}{\sqrt{x+1} + \sqrt{x-1}} dx.$

(iii) $\int \frac{1}{\sin(x-a)\sin(x-b)} dx.$ (iv) $\int \frac{\cos^2 x}{1+\sin x} dx$

(v) $\int \cos x \cos 2x \cos 3x \; dx.$ (vi) $\int \cos^5 x \; dx$

(vii) $\int \frac{\sin x \cos x}{\sqrt{a^2 \sin^2 x + b^2 \cos^2 x}} dx.$ (viii) $\int \frac{1}{\sqrt{\cos^3 x \cos(x+a)}} dx.$

(ix) $\int \frac{\operatorname{Sin}^6 x + \cos^6 x}{\sin^2 x \cos^2 x} dx.$

2. Evaluate :

(i) $\int \frac{x dx}{x^4 + x^2 + 1}.$ *(ii) $\int \frac{1/x \quad dx}{\left[6(\log x)^2 + 7\log x + 2\right]}$

(iii) $\int \frac{dx}{1 + x - x^2}.$ (iv) $\int \frac{1}{\sqrt{9 + 8x - x^2}} dx.$

(v) $\int \frac{1}{\sqrt{(x-a)(x-b)}} dx.$ (vi) $\int \sqrt{\frac{\sin(x-\alpha)}{\sin(x+\alpha)}} dx.$

(vii) $\int \frac{5x - 2}{3x^2 + 2x + 1} dx.$ (viii) $\int \frac{x^2}{x^2 + 6x + 12} dx.$

(ix) $\int \frac{x+2}{\sqrt{4x - x^2}} dx.$ (x) $\int x\sqrt{1 + x - x^2} dx.$

(xii) $\int (3x - 2)\sqrt{x^2 + x + 1} \; dx.$ (xiii) $\int \sqrt{\sec x + 1} \; dx.$

3. Evaluate :

(i) $\displaystyle\int \frac{dx}{x\left(x^7+1\right)}$.

(ii) $\displaystyle\int \frac{\sin x}{(1+\cos x)(2+3\cos x)}\,dx$.

(iii) $\displaystyle\int \frac{\sin\theta\cos\theta}{\cos^2\theta-\cos\theta-2}\,d\theta$.

(iv) $\displaystyle\int \frac{x-1}{(x+1)(x-2)(x+3)}\,dx$.

(v) $\displaystyle\int \frac{x^2+x+2}{(x-2)(x-1)}\,dx$.

(vi) $\displaystyle\int \frac{\left(x^2+1\right)\left(x^2+2\right)}{\left(x^2+3\right)\left(x^2+4\right)}\,dx$.

(vii) $\displaystyle\int \frac{dx}{(2x+1)\left(x^2+7\right)}$

(viii) $\displaystyle\int \frac{dx}{\sin x\,(1-2\cos x)}$

(ix) $\displaystyle\int \frac{\sin x}{\sin 4x}\,dx$.

(x) $\displaystyle\int \sqrt{\tan x}\;dx$.

4. Evaluate :

(i) $\displaystyle\int x^5 \sin x^3\,d\,x$.

(ii) $\displaystyle\int \sec^3 x\,dx$.

(iii) $\displaystyle\int e^{ax}\cos(bx+c)\,dx$.

(iv) $\displaystyle\int \sin^{-1}\frac{6x}{1+9x^2}\,dx$.

(v) $\displaystyle\int \cos\sqrt{x}\,dx$.

(vi) $\displaystyle\int x^3\tan^{-1}x\;dx$.

(vii) $\displaystyle\int e^{2x}\left(\frac{1+\sin 2x}{1+\cos 2x}\right)dx$.

(viii) $\displaystyle\int e^{x}\left(\frac{x-1}{2x^2}\right)dx$.

(ix) $\displaystyle\int e^{x}\left(\frac{1-x}{1+x^2}\right)^2 dx$.

(x) $\displaystyle\int e^{x}\frac{\left(x^2+1\right)}{(x+1)^2}\,dx$.

(xi) $\displaystyle\int e^{x}\frac{(2+\sin 2x)}{(1+\cos 2x)}\,dx$.

(xii) $\displaystyle\int \left\{\log(\log x)+\frac{1}{(\log x)^2}\right\}dx$.

5. Evaluate the following definite integrals :

(i) $\int_0^{\frac{\pi}{4}} \frac{\sin x + \cos x}{9 + 16 \sin 2x} dx.$ (ii) $\int_0^{\frac{\pi}{2}} \cos 2x \log \sin x \ dx.$

(iii) $\int_0^2 \frac{x + \sin x}{1 + \cos x} dx.$

6. Evaluate :

(i) $\int_1^3 \{|x - 1| + |x - 2| + |x - 3|\} dx.$ (ii) $\int_0^{\pi} \frac{x}{1 + \sin x} dx.$

(iii) $\int_0^{\frac{\pi}{4}} \log (1 + \tan x) dx.$ (iv) $\int_0^{\frac{\pi}{2}} \log \sin x \ dx.$

(v) $\int_0^{\pi} \frac{x \sin x}{1 + \cos^2 x} dx.$

(vi) $\int_{-2}^{2} f(x) dx$ where $f(x) = \begin{cases} 2x - x^3 & \text{when } -2 \leq x < -1 \\ x^3 - 3x + 2 & \text{when } -1 \leq x < 1 \\ 3x - 2 & \text{when } 1 \leq x < 2. \end{cases}$

7. Evaluate the following integrals

(i) $\int_1^3 \left|x^2 - 2x\right| dx.$

(ii) $\int_0^1 \sin^{-1}\left(\frac{2x}{1+x^2}\right) dx.$

(iii) $\int_{-1}^1 \log\left(\frac{1+\sin x}{1-\sin x}\right) dx.$

(iv) $\int_0^\pi \frac{e^{\cos x}}{e^{\cos x} + e^{-\cos x}}\ dx.$

8. Evaluate the following integrals :

(iii) $\int \frac{2x^3}{(x+1)(x-3)} dx$

(iv) $\int \frac{x^4}{x^4 - 16} dx$

(v) $\int_0^2 \left(\sqrt{\tan x} + \sqrt{\cot x}\right) dx.$

(vi) $\int \frac{1}{x^4 + 1} dx.$

(vii) $\int_0^\infty \frac{x \tan^{-1} x}{\left(1+x^2\right)^2} dx.$

9. Evaluate the following integrals as limit of sums :

(i) $\int_2^4 (2x + 1) dx.$

(ii) $\int_0^2 \left(x^2 + 3\right) dx.$

(iii) $\int_0^4 \left(3x^2 + e^{2x}\right) dx.$

10. Evaluate the following integrals :

(i) $\int \frac{\sqrt{x^2+1}\left[\log\left(x^2+1\right)-2\log x\right]}{x}\,dx$ (ii) $\int \frac{x^2}{\left(x\sin x+\cos x\right)^2}\,dx$

(iii) $\int \sin^{-1}\sqrt{\frac{x}{a+x}}\,dx$ (iv) $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sin x+\cos x}{\sqrt{\sin 2x}}\,dx$

(iv) $\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \left(\sin|x|-\cos|x|\right)dx$ (vi) $\int_{\frac{-\pi}{4}}^{\frac{\pi}{2}} |\sin x|\,dx$

(vii) $\int_{0}^{1.5} \left[x^2\right]dx$ where [x] is greatest integer function

(viii) $\int_{-1}^{\frac{3}{2}} |x\sin \pi x|\,dx$

CHAPTER-8

APPLICATIONS OF INTEGRALS

(4 Mark Questions)

1. Find the area enclosed by circle $x^2 + y^2 = a^2$.
2. Find the area of region bounded by $y^2 = 4x$.
3. Find the area enclosed by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$
4. Find the area of region in the first quadrant enclosed by x–axis the line y = x and the circle $x^2 + y^2 = 32$.
5. Find the area of region $\{(x, y) : y^2 \leq 4x,\ 4x^2 + 4y^2 \leq 9\}$
6. Prove that the curve $y = x^2$ and, $x = y^2$ divide the square bounded by x = 0, y = 0, x = 1, y = 1 into three equal parts.

7 Find area enclosed between the curves, y = 4x and $x^2 = 6y$.

8. Find the common area bounded by the circles $x^2 + y^2 = 4$ and $(x - 2)^2 + y^2 = 4$.

9. Using integration, find the area of the region bounded by the triangle whose vertices are

   (a) (–1, 0), (1, 3) and (3, 2)   (b) (–2, 2) (0, 5) and (3, 2)

10. Using integration, find the area bounded by the lines.

    (i) $x + 2y = 2$, $y - x = 1$ and $2x + y - 7 = 0$

    (ii) $y = 4x + 5$, $y = 5 - x$ and $4y - x = 5$.

11. Find the area of the region $\{(x, y) : x^2 + y^2 \leq 1 \leq x + y\}$.

12. Find the area of the region bounded by

    $y = |x - 1|$ and $y = 1$.

13. Find the area enclosed by the curve $y = \sin x$ between $x = 0$ and $x = \frac{3\pi}{2}$ and x-axis.

14. Find the area bounded by semi circle $y = \sqrt{25 - x^2}$ and x-axis.

15. Find area of region given by $\{(x, y) : x^2 \leq y \leq |x|\}$.

16. Find area of smaller region bounded by ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ and straight line $2x + 3y = 6$.

17. Find the area of region bounded by the curve $x^2 = 4y$ and line $x = 4y - 2$.

18. Using integration find the area of region in first quadrant enclosed by x-axis the line $x = \sqrt{3}y$ and the circle $x^2 + y^2 = 4$.

19. Find smaller of two areas bounded by the curve $y = |x|$ and $x^2 + y^2 = 8$.

20. Find the area lying above x-axis and included between the circle $x^2 + y^2 = 8x$ and the parabola $y^2 = 4x$.

21. Using integration, find the area enclosed by the curve $y = \cos x$, $y = \sin x$ and x-axis in the interval $\left(0, \frac{\pi}{2}\right)$.

22. Sketch the graph $y = |x - 5|$. Evaluate $\int_0^6 |x - 5|\, dx$.

CHAPTER-9

DIFFERENTIAL EQUATION

(1 Mark Questions)

1. The degree of the differential equation.

$$\left(\frac{d^2y}{dx^2}\right)^3+\left(\frac{dy}{dx}\right)^2+\sin\left(\frac{dy}{dx}\right)+1=0 \text{ is}$$

(A) 3 (B) 2 (C) 0 (D) not defined

2. The differential equation

$$y=x\frac{dy}{dx}+\frac{1}{dy/dx} \text{ is of}$$

(A) order 2 and degree 1

(B) order 1 and degree 2

(C) order 1 and degree 1

(D) order 2 and degree 2

3. The order of the differential equation

$$\left(\frac{d^2s}{dt^2}\right)^2+3\left(\frac{ds}{dt}\right)^3+4=0 \text{ is}$$

(A) 1 (B) 2 (C) 3 (D) 4

4. The order of the differential equation.

$$\frac{d^2y}{dx^2}=\sqrt{1+\left(\frac{dy}{dx}\right)^3} \text{ is}$$

(A) 2 (B) 1 (C) 3 (D) None of these

5. The solution of $\frac{d^2y}{dx^2}=0$ presents :

(A) a st. line (B) a circle

(C) a parabola (D) a point

6. The integrating factor of the differential equation.

$$x\frac{dy}{dx} - y = 2x^2 \text{ is.}$$

(A) e^{-x} (B) e^{-y} (C) $\frac{1}{x}$ (D) x

7. The order of the differential equation :

$$\left\{1+\left(\frac{dy}{dx}\right)^2\right\}^{3/2} = \frac{d^3y}{dx^3} \text{ is}$$

(A) 1 (B) 2 (C) 3 (D) 4

8. The order and degree of diff. equ.

$$\left(1+3\frac{dy}{dx}\right)^{2/3} = 4\frac{d^3y}{dx^3} \text{ are}$$

(A) $\left(1,\frac{2}{3}\right)$ (B) (3,1) (C) (3,3) (D) (1,2)

9. The solution of the differential equ.

$$ydx+\left(x+x^2y\right)dx=0 \text{ is}$$

(A) $\frac{1}{xy}+\log y=c$ (B) $-\frac{1}{xy}+\log y=c$

(C) $-\frac{1}{xy}=c$ (D) $\log y = cx$

10. A solution of the differential equation

$$\left(\frac{dy}{dx}\right)^2 - x\left(\frac{dy}{dx}\right)+y=0 \text{ is}$$

(A) $y = 2$ (B) $y = 2x$

(C) $y = 2x-4$ (D) $y=2x^2-4$

11. If $y^1 = \frac{x-y}{x+y}$ then, its solution is :

(A) $y^2+2xy-x^2=c$ (B) $y^2+2xy+x^2=c$

(C) $y^2-2xy-x^2=c$ (D) $y^2-2xy+x^2=c$

12. Solution of $\frac{dy}{dx}+2xy=y$ is

(A) $y=ce^{x-x^2}$ (B) $y=ce^{x^2-x}$

(C) $y=ce^x$ (D) None of the above.

13. The general solution of $\frac{dy}{dx}+2x=e^x$ is

(A) $y=\frac{1}{3}e^x+ce^{-2x}$ (B) $y=e^x+x^2+c$

(C) $y=-e^x+x^2+c$ (D) $y=e^x+c$

14. The solution of the differential equation.

$3e^x \tan y\, dx+(1-e^x)\sec^2 y\, dy=0$ is

(A) $\cot y=c(1-e^x)^3$ (B) $\tan y=c(1-e^x)^3$

(C) $\tan y=c(1+e^x)^3$ (D) None of these

(2 Mark Questions)

Form the differential equations of the family of curves (1-4)

1. $y=e^x(A\cos x+B\sin x)$, where A and B are arbitrary constants

2. $y=C(x-c)^2$ where C is arbitrary constant

3. $y = ke^{\sin^{-1} x} + 3$ where k is arbitrary constant.

4. Form the differential equation of the family of circles touching y-axis at (0, 0).

5. Solve the differential eqn.

$$\frac{dy}{dx} = 1 + x + y + xy$$

6. Solve $\frac{dy}{dx} = e^{-y} \cos x$, given that y(0)=0

7. Solve $\frac{dy}{dx} = \frac{1 + y^2}{1 + x^2}$

8. Solve $\frac{dy}{dx} = \frac{2x^3 y}{(x+1)}$

9. Find the general solution of the following

$$\left(e^x + e^{-x}\right)\frac{dy}{dx} = \left(e^x - e^{-x}\right)$$

10. Solve the differential eqn.

$\frac{dy}{dx} = x^2 e^{-3y}$ given that y=0 for x=0

11. Solve the differential eqn.

$\left(1 + e^{2x}\right) dy + \left(1 + y^2\right) e^x dx = 0$ given that x=0, y=1

(4 **Mark Questions**)

1. Show that the differential equation $\frac{dy}{dx} = \frac{x + 2y}{x - 2y}$ is homogeneous and solve it.

2. Show that the differential equation :

$(x^2 + 2xy - y^2)\, dx + (y^2 + 2xy - x^2)\, dy = 0$ is homogeneous and solve it.

3. Solve the following differential equations :

$$\sin x \frac{dy}{dx} + y \cos x = 2 \sin^2 x \cos x \text{ if } y\left(\frac{\pi}{2}\right) = 1$$

Solve the following differential equations :

4. $(x^3 + y^3)\,dx = (x^2y + xy^2)dy.$

5. $y\left\{x\cos\left(\frac{y}{x}\right) + y\sin\left(\frac{y}{x}\right)\right\}dx - x\left\{y\sin\left(\frac{y}{x}\right) - x\cos\left(\frac{y}{x}\right)\right\}dy = 0.$

6. $x^2dy + y(x + y)\,dx = 0$ given that $y = 1$ when $x = 1$.

7. $xe^{\frac{y}{x}} - y + x\frac{dy}{dx} = 0$ if $y(e) = 0$

8. $(x^3 - 3xy^2)\,dx = (y^3 - 3x^2y)dy.$

Solving the following differential equation

9. $\cos^2 \frac{dy}{dx} = \tan x - y.$

10. $x\cos x\frac{dy}{dx} + y\,(x\sin x + \cos x) = 1.$

11. $\left(1 + e^{\frac{x}{y}}\right)dx + e^{\frac{x}{y}}\left(1 - \frac{x}{y}\right)dy = 0.$

12. $(y - \sin x)\,dx + \tan x\,dy = 0,\ y(0) = 0.$

13 $3e^x \tan y\,dx + (1 - e^x)\sec^2 y\,dy = 0$ given that $y = \frac{\pi}{4}$, when $x = 1$.

14 $\frac{dy}{dx} + y\cot x = 2x + x^2\cot x$ given that $y(0) = 0$.

CHAPTER-10

VECTORS

(1 Mark Questions)

1. If θ is the angle between two vectors $\vec{a},\vec{b}$ then $\vec{a}.\vec{b} \geq 0$ only when

(A) $0 < \theta < \frac{\pi}{2}$ (B) $0 \leq \theta \leq \frac{\pi}{2}$

(C) $0 < \theta < \pi$ (D) $0 \leq \theta \leq \pi$

2. The vector $2\hat{i} + \hat{j} - \hat{k}$ and $4\hat{i} + 2\hat{j} + 10\hat{k}$ are

(A) at angle of $\frac{\pi}{3}$ (B) of equal magnitude

(C) Parallel (D) orthogonal

3. The projection of the vector $\hat{i} - 2\hat{j} + \hat{k}$ on the vector $4\hat{i} - 4\hat{j} + 7\hat{k}$ is

(A) $\frac{5\sqrt{5}}{19}$ (B) $2\frac{1}{9}$ (C) $\frac{9}{19}$ (D) $\frac{\sqrt{6}}{19}$

4. If $\vec{a}$ and $\vec{b}$ are two collinear vectors, then which of the following are incorrect:

(A) $\vec{b} = \lambda\vec{a}$ for some scalar λ

(B) $\vec{a} = \pm\vec{b}$

(C) The respective components of $\vec{a}$ and $\vec{b}$ are proportional

(D) Both the vector $\vec{a}$ and $\vec{b}$ have the same direction, but different magnitude.

5. If $\hat{i}, \hat{j}, \hat{k}$ have the usual meaning in vectors, then $\hat{i}.\hat{j} = \hat{j}.\hat{k} = \hat{k}.\hat{i}$ is

(A) -1 (B) 0 (C) 1 (D) None of these.

6. The unit vector perpendicular to the vector $\hat{i}+\hat{j}$ and $\hat{j}+\hat{k}$ are

(A) $\hat{i}+\hat{j}+\hat{k}$

(B) $\frac{1}{\sqrt{3}}\left(\hat{i}+\hat{j}+\hat{k}\right)$

(C) $\hat{i}-\hat{j}+\hat{k}$

(D) $\frac{1}{\sqrt{3}}\left(\hat{i}-\hat{j}+\hat{k}\right)$

7. If $\vec{a}.\vec{b}=\vec{b}.\vec{c}=\vec{c}.\vec{a}=0$, then $\vec{a}$ is equal to

(A) a non zero vector (B) 1

(C) - 1 (D) $|\vec{a}||\vec{b}||\vec{c}|$

8. The vector $\vec{a}=3\hat{i}-\hat{k},\vec{b}=\hat{i}+2\hat{j}$ are adjacent sides of a parallelogram. Its area is

(A) $\frac{1}{2}\sqrt{17}$ (B) $\frac{1}{2}\sqrt{14}$

(C) $\sqrt{41}$ (D) $\frac{1}{2}\sqrt{17}$

9. The vector $2\hat{i}+\hat{j}-\hat{k}$ is perpendicular $\hat{i}-4\hat{j}-\lambda\hat{k}$ iff λ equals.

(A) 0 (B) -1 (C) 2 (D) -3

10. If $|\vec{a}+\vec{b}|=|\vec{a}-\vec{b}|$, then the vectors $\vec{a}$ and $\vec{b}$ are

(A) parallel (b) perpendicular

(c) inclined at angle $\frac{\pi}{4}$ (d) inclined at an angle $\frac{\pi}{6}$

11. The quantity $\left(\vec{a}\times\vec{b}\right).\left(\vec{c}\times\vec{d}\right)$ is :

(A) not defined (b) vector

(c) scalar (d) nature depends upon $\vec{a},\vec{b},\vec{c},\vec{d}$

12. If $|\vec{a}|=|\vec{b}|$, then $(\vec{a}+\vec{b}).(\vec{a}-\vec{b})$ is

(A) zero (b) negative

(c) positive (d) none of these

13. If $\vec{a}.\vec{b}=|\vec{a}||\vec{b}|$, then $\vec{a}$ and $\vec{b}$ are

(A) perpendicular (b) like parallel

(c) unlike parallel (d) coincident

14. The area of the triangle whose adjacent sides are :

$\vec{a}=3\hat{i}+\hat{j}+4\hat{k}$ and $\vec{b}=\hat{i}-\hat{j}+\hat{k}$ is

(A) $\sqrt{42}$ (B) $\sqrt{\frac{42}{2}}$

(C) $\frac{\sqrt{42}}{2}$ (D) $\frac{2}{\sqrt{42}}$

15. The vectors $3\hat{i}+4\hat{j}-6\hat{k}$ and $-6\hat{i}+8\hat{j}+12\hat{k}$ are

(A) equal (B) of same magnitude

(C) parallel (D) mutually perpendicular

16. The work done is moving an object along a vector $\vec{d}=3\hat{i}+2\hat{j}-5\hat{k}$, if the applied force is $\vec{F}=2\hat{i}-\hat{j}-\hat{k}$,is

(A) 12 units (B) 11 units

(C) 10 units (D) 9 units

(4 Mark Questions)

1. If ABCDEF is a regular hexagon, then using triangle law of addition, prove that

$$\overrightarrow{AB}+\overrightarrow{AC}+\overrightarrow{AD}+\overrightarrow{AE}+\overrightarrow{AF}=3\overrightarrow{AD}=6\overrightarrow{AO}$$

2. The scalar product of vector $\hat{i}+\hat{j}+\hat{k}$ with unit vector along the sum of the vectors $2\hat{i}+4\hat{j}-5\hat{k}$ and $\lambda\hat{i}+2\hat{j}+3\hat{k}$ is equal to 1. Find the value of λ.

3. $\vec{a},\vec{b}$ and $\vec{c}$ are three mutually perpendicular vectors of equal magnitude. Show that $\vec{a}+\vec{b}+\vec{c}$ makes equal angles with $\vec{a},\vec{b}$ and $\vec{c}$ with each angle as $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$

4. If $\vec{\alpha}=3\vec{i}-\vec{j}$ and $\vec{\beta}=2\vec{i}+\vec{j}+3\vec{k}$, then express $\vec{\beta}$ in the form of $\vec{\beta}=\vec{\beta}_1+\vec{\beta}_2$, where $\vec{B}_1$ is parallel to $\vec{\alpha}$ and $\vec{\beta}_2$ is perpendicular to $\vec{\alpha}$.

5. If $\vec{a},\vec{b},\vec{c}$ are three vectors such that $\vec{a}+\vec{b}+\vec{c}=0$, then prove that $\vec{a}\times\vec{b}=\vec{b}\times\vec{c}=\vec{c}\times\vec{a}$.

6. If $|\vec{a}|=3, |\vec{b}|=5, |\vec{c}|=7$ and $\vec{a}+\vec{b}+\vec{c}=\vec{0}$, find the angle between $\vec{a}$ and $\vec{b}$.

7. If $\vec{a}=\hat{i}+\hat{j}+\hat{k}, \vec{c}=\hat{j}-\hat{k}$ are the given vectors then, find a vector $\vec{b}$ satisfying the equation $\vec{a}\times b=\vec{c}$ and $\vec{a}.\vec{b}=3$.

8. For any two vector, $|\vec{a}+\vec{b}|\le|\vec{a}|+|\vec{b}|$

9. For any two vector, $|\vec{a}\times\vec{b}|=\sqrt{a^2b^2-\left(\vec{a}.\vec{b}\right)^2}$

10. Prove that the angle between any two diagonals of a cube is $\cos^{-1}\left(\frac{1}{3}\right)$.

11. Let $\vec{a},\vec{b}$ and $\vec{c}$ are unit vectors such that $\vec{a}.\vec{b}=\vec{a}.\vec{c}=\vec{0}$ and angle between $\vec{b}$ and $\vec{c}$ is $\frac{\pi}{6}$, then prove that $\vec{a}=\pm 2\left(\vec{b}\times\vec{c}\right)$.

12. Prove that the normal vector to the plane containing three points with position vectors $\vec{a},\vec{b}$ and $\vec{c}$ lies in the direction of vector $\vec{b}\times\vec{c}+\vec{c}\times\vec{a}+\vec{a}\times\vec{b}$.

13. If $\vec{a},\vec{b},\vec{c}$ are position vectors of the vertices A, B, C of a triangle ABC, than show the area of ΔABCis $\frac{1}{2}\left|\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}\right|$

14. If $\vec{a}=5\hat{i}-\hat{j}+7\hat{k}, \vec{b}=\hat{i}-\hat{j}-\lambda\hat{k}$, find λ such that $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$ are orthogonal.

15. Let $\vec{a}$ and $\vec{b}$ be vectors such that $|\vec{a}|=|\vec{b}|=|\vec{a}-\vec{b}|=1$, find $|\vec{a}+\vec{b}|$

16. If $|\vec{a}|=2, |\vec{b}|=5, \vec{a}\times\vec{b}=2\vec{i}+\vec{j}-2\vec{k}$ find the value of $\vec{a}.\vec{b}$

CHAPTER-11

THREE DIMENSIONAL GEOMETRY

(1 Mark Questions)

1. If the direction cosines of a line are <k,k,k> then

   (A) k>0 (B) 0<k<1 (C) k=1 (D) $k=\frac{1}{\sqrt{3}}$ or $-\frac{1}{\sqrt{3}}$

2. Distance of the point (α,β,γ) from the XOY-plane is

   (A) γ (B) $|\gamma|$ (C) $\sqrt{\alpha^2+\beta^2}$ (D) None of these.

3. The distance of the plane $\vec{r}.\left(\frac{2}{3}\hat{i}+\frac{3}{7}\hat{j}-\frac{6}{7}\hat{k}\right)=1$ from the origin is

   (A) 1 (B) 7 (C) $\frac{1}{7}$ (D) none of these

4. The lines $\frac{x-1}{1}=\frac{y-1}{2}=\frac{z-3}{0}$ and $\frac{x-2}{0}=\frac{y-3}{0}=\frac{z-4}{1}$ are

   (A) parallel (B) skew

   (C) coincident (D) perpendicular

5. The distance between the planes :

   $3x+2y-6z-14=0$ and $3x+2y-6z+21=0$ is

   (A) 35 (B) 7

   (C) 1 (D) 5

6. The line $\frac{x-x_1}{0}=\frac{y-y_1}{1}=\frac{z-z_1}{2}$ is

(A) at right angles to *x*-axis.

(B) at right angles to the plane YOZ

(C) is parallel to y-axis

(D) none of these.

7. The points (0,0,0), (2,0,0), $\left(1,\sqrt{3},0\right)$ and $\left(1,\frac{1}{\sqrt{3}},\frac{2\sqrt{2}}{\sqrt{3}}\right)$ are the vertices of a

(A) square (B) rhombus

(C) rectangle (D) regular tetrahedron

8. The plane $x-2y+z-6=0$ and the $x-2y+z-6=0$ are related as

(a) parallel to the line

(b) at right angles to the plane.

(c) lines in the plane

(D) meets the plane obliquely.

9. The plane containing the point (3,2,0) and the line $\frac{x-3}{1}=\frac{y-6}{5}=\frac{z-4}{4}$ is

(A) $x-y+z=1$ (B) $x+y+z=5$

(C) $x+2y-z=1$ (D) $2x-y+z=5$

10. The line $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$ and the plane $x-2y+z=0$ are related as the line

(A) meets the plane in a unique point

(B) lines in the plane

(C) meets the plane at right angles

(D) is parallel to the plane.

11. The sine of the angle between the straight line $\frac{x-2}{3}=\frac{y-3}{4}=\frac{z-4}{5}$ and the plane $2x-2y+2=5$ is

(A) $\frac{10}{6\sqrt{5}}$ (B) $\frac{4}{5\sqrt{2}}$

(C) $\frac{2\sqrt{3}}{5}$ (D) $\frac{\sqrt{2}}{10}$

12. The reflection of the point (α,β,γ) in the XOY plane is

(A) (α,β,o) (B) (o,o,γ)

(C) $(-\alpha,-\beta,\gamma)$ (D) $(\alpha,\beta,-\gamma)$

13. The projection of the point (1,2,-4) in the YOZ plane is.

(A) (0,2,-4) (B) (1,0,0)

(C) (-1,2,-4) (D) (1,2,4)

(2 Mark Questions)

1. Find the direction – cosines of a line, which makes equal angles with the co-ordinate axes.

2. Find the acute angle between two lines whose direction-ratios are <2,3,6> and <1,2,-2>.

3. Let α,β,γ are direction angles of a line. Prove that cos2α+cos2β + cos 2γ +1=0

4. The cartesian equations of a line are :

$$3x + 1 = 6y - 2 = 1 - z$$

Find the direction – ratios and write its equation in vector form.

5. Find the equation of the Plane with intercept 3 on the y-axis and parallel to zox plane.

6. Find the value of λ so that the Planes : $2x+\lambda y+3z=15$ and $x-y+7\lambda y=13$ are perpendicular.

7. Find the point of intersection of the line : $r=\left(\hat{i}+2\hat{j}+3\hat{k}\right)+\lambda\left(2\hat{i}+\hat{j}+2\hat{k}\right)$ and the plane $r.\left(2\vec{i}-6\hat{j}+3\hat{k}\right)+5=0$

8. Find the vector equation of a Plane, which is at a distance of 6 units from origin and which is normal to the vector $2\vec{i}-\hat{j}+2\hat{k}$.

9. Find the angle between two planes $2x+y-2z=5$ and $3x-6y-2z=7$

10. The Cartesian and Vector equations of a line, which passes through the point (1,2,3) and is parallel to the line $\dfrac{-x-2}{1}=\dfrac{y+3}{7}=\dfrac{2z+6}{3}$

(6 Mark Questions)

1) Find shortest distance between the lines

$$\frac{x-8}{3}=\frac{y+19}{-16}=\frac{z-10}{7} \text{ and } \frac{x-15}{3}=\frac{y-29}{8}=\frac{z-5}{-5}$$

2) Find shortest distance between the lines

$$\vec{r}=(1-\lambda)\hat{i}+(\lambda-2)\hat{j}+(3-2\lambda)\hat{k}$$

and $$\vec{r}=(\mu+1)\hat{i}+(2\mu-1)\hat{j}+(2\mu+1)\hat{k}$$

3) A variable plane is at a constant distance 3p from the origin and meet the co-ordinate axes in A, B and C respectively. Show that the locus of centroid of ΔABC is

$$x^{-2}+y^{-2}+z^{-2}=p^{-2}$$

4) Find the foot of perpendicular from the point $2\hat{i}-\hat{j}+5\hat{k}$ on the line $\vec{r}=\left(11\hat{i}-2\hat{j}-8\hat{k}\right)+\lambda\left(10\hat{i}-4\hat{j}-11\hat{k}\right)$. Also, find the length of perpendicular.

5) A line makes angle $\alpha,\beta,\gamma,\delta$ with a four diagonals of a cube. Prove that

$$\cos^2\alpha+\cos^2\beta+\cos^2\gamma+\cos^2\delta=\frac{4}{3}$$

6) Find the point of intersection of the lines

$$\frac{x+1}{-3}=\frac{y-3}{2}=\frac{z+2}{1} \text{ and } \frac{x}{1}=\frac{y-7}{-3}=\frac{z+7}{2}$$

Also, find the equation of the Plane in which they lie.

7) Find the equ. of the plane passing through the intersection of planes $2x+3y-z=-1$ and $x+y-2z+3=0$ and perpendicular to the plane $3x-y-2z=4$. Also, find the inclination of this plane with *XY*–plane.

8) Prove that the image of the point (3,-2,1) in the plane $3x-y+4z=2$ lies in the plane $x+y+z+4=0$.

9) Find the equations of the two lines through the origin such that each line is intersecting the line $\frac{x-3}{2}=\frac{y-3}{1}=\frac{z}{1}$ at an angle of $\frac{\pi}{3}$.

10) Find the equ. of plane containing the parallel lines

$$\frac{x-4}{1}=\frac{y-3}{-4}=\frac{z-2}{5} \text{ and } \frac{x-3}{1}=\frac{y+2}{-4}=\frac{z}{5}$$

11) prove that if a plane has the intercept a,b and c and is at a distance of *p* units from the origin, then

$$\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}=\frac{1}{p^2}$$

12) Find the vector equation of the line passing through the point (1,2,-4) and perpendicular to the two lines

$$\frac{x-8}{3}=\frac{y+19}{-16}=\frac{z-10}{7} \text{ and } \frac{x-15}{3}=\frac{y-29}{8}=\frac{z-5}{-5}$$

13. Find the coordinate of the foot of the perpendicular and the perpendicular distance of the point (1,3,4) from the plane $2x-y+Z+3=0$ Find also the image of the Point in the plane.

CHAPTER 12
LINEAR PROGRAMMING

(6 Mark Questions)

1. Solve the following L.P.P. graphically

   Minimise and maximise $z = 3x + 9y$

   Subject to the constraints $x + 3y \leq 60$

   $x + y \geq 10$

   $x \leq y$

   $x \geq 0,\ y \geq 0$

2. Determine graphically the minimum value of the objective function $z = -50x + 20y$

   Subject to the constraints $2x - y \geq -5$

   $3x + y \geq 3$

   $2x - 3y \leq 12$

   $x \geq 0,\ y \geq 0$

3. Maximize $z = 11x + 5y$

   subject to the constraints :

   $3x + 2y \leq 25,\ x + y \leq 10,\ x, y \geq 0$

4. Maximize $z = 10x + 12y$

   subject to the constraints :

   $2x + 3y \leq 30,\ 3x + y \leq 17,\ x, y \geq 0$

5. Minimize $z = -3x + 4y$

   subject to the constraints :

   $x + 2y \leq 8,\ 3x + 2y \leq 12,\ x \geq 0,0,\ y \geq 0$

6. Minimize $z = 2x + 3y$

subject to constraints :

$x \geq 0, y \geq 0, \ 1 \leq x + 2y \leq 0$

7. Maximize and minimize $z = 5x + 10y$

subject to constraints :

$x + 2y \leq 120, \ x + y \geq 60,$

$x - 2y \geq 0, \ x \geq 0, y \geq 0$

8. Maximize and Minimize $z = x + 2y$

subject to constraints :

$x + 2y \geq 100, 2x - y \leq 0,$

$2x + y \leq 200, \ x, y \geq 0$

9. Minimize $z = 3x + 2y$

subject to the constraints

$x + y \geq 8, 3x + 5y \leq 15, x \geq 0, y \geq 0$

10. Minimize $z = 5x + 3y$

Subject to the constraints

$2x + y \geq 10, x + 3y \geq 15$

$x \leq 10, \ y \leq 8, \ x, y \geq 0$

11. Two tailors A and B earn Rs. 150 and Rs. 200 per day respectively. A can stitch 6 shirts and pants per day, while B can stich 10 shirts and 4 pants per day. Formulate the above L.P.P. mathematically and hence solve it to minimise the labour cost to produce at least 60 shirts and 32 pants.

12. There are two types of fertilisers A and B. A consists of 10% nitrogen and 6% phosphoric acid and B consists of 5% nitrogen and 10% phosphoric acid. After testing the soil conditions, a farmer finds that he needs at least 14 kg of nitrogen and 14 kg of phosphoric acid for his crop. If A costs Rs. 61 kg and B costs Rs. 51 kg, determine how much of each type of fertiliser should be used so that nutrient requirements are met at minimum cost. What is the minimum cost.

13. A man has Rs. 1500 to purchase two types of shares of two different companies S_1 and S_2.
Market price of one share of S_1 is Rs 180 and S_2 is Rs. 120. He wishes to purchase a maximum to ten shares only. If one share of type S_1 gives a yield of Rs. 11 and of type S_2 Rs. 8 then how much shares of each type must be purchased to get maximum profit? And what will be the maximum profit?

14. A company manufacture two types of lamps say A and B. Both lamps go through a cutter and then a finisher. Lamp A requires 2 hours of the cutter's time and 1 hours of the finisher's time. Lamp B requires 1 hour of cutter's and 2 hours of finisher's time. The cutter has 100 hours and finishers has 80 hours of time available each month. Profit on one lamp A is Rs. 7.00 and on one lamp B is Rs. 13.00. Assuming that he can sell all that he produces, how many of each type of lamps should be manufactured to obtain maximum profit?

15. A dealer wishes to purchase a number of fans and sewing machines. He has only Rs. 5760 to invest and has space for almost 20 items. A fan and sewing machine cost Rs. 360 and Rs. 240 respectively. He can sell a fan at a profit of Rs. 22 and sewing machine at a profit of Rs. 18. Assuming that he can sell whatever he buys, how should he invest his money to maximise his profit?

16. If a young man rides his motorcycle at 25 km/h, he has to spend Rs. 2 per km on petrol. If he rides at a faster speed of 40 km/h, the petrol cost increases to Rs. 5 per km. He has Rs. 100 to spend on petrol and wishes to find the maximum distance he can travel within one hour. Express this as L.P.P. and then solve it graphically.

17. A producer has 20 and 10 units of labour and capital respectively which he can use to produce two kinds of goods X and Y. To produce one unit of X, 2 units of capital and 1 unit of labour is required. To produce one unit of Y, 3 units of labour and one unit of capital is required. If X and Y are priced at Rs. 80 and Rs. 100 per unit respectively, how should the producer use his resources to maximise the total revenue?

CHAPTER – 13

"PROBABILITY"

(1 Mark Questions)

1. If $P(A)=\frac{1}{2}$, $P(B)=0$, than $P(A/B)$ is

(a) 0 (b) $\frac{1}{2}$ (c) Not defined (d) 1

2. Two dice are thrown simultaneously. The probability of getting six as a product is:

(a) $\frac{1}{9}$ (b) $\frac{2}{9}$ (c) $\frac{4}{9}$ (d) $\frac{5}{9}$

3. For any two events A and B. $P(\overline{A}\cup\overline{B})$ is always equal to :

(a) $P(\overline{A})+P(\overline{B})$ (b) $P(\overline{A})P(\overline{B})$

(c) $1-P(A\cup B)$ (d) $1-P(A\cap B)$

4. If $P(A)=\frac{1}{5}, P(B)=\frac{2}{5}$ and $P(A\cup B)=\frac{3}{5}$, then P (A/B) is

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) 0 (d) $\frac{1}{2}$

5. If X and Y are two independent events, then P(X and Y) is equal to

(a) P(X) + P(Y) (b) P(X) P(Y)

(c) P(X) +P(Y) – P(X or Y) (d) None of these

6. In a single throw of a pair of dice, the probability of getting doublets of odd numbers is :

(a) $\frac{1}{12}$ (b) $\frac{1}{6}$ (c) $\frac{1}{4}$ (d) $\frac{1}{9}$

7. If A and B are events such that P(A/B) = P(B/A) then :

(a) ACB, but A $\neq$ B (b) A=B

(c) $A \cap B = \phi$ (d) P(A)=P(B)

8. Form a deck of 52 cards, the probability of drawing a Heart card is

(a) $\frac{4}{3}$ (b) $\frac{1}{4}$ (c) $\frac{1}{3}$ (d) None of these

9. Two coins are tossed four times. The number of elements in sample space is :

(a) 8 (b) 4 (c) 16 (d) 36

10. A and B are two independent events such that $P(A \cup B) = 0.8$ and P(A)=0.3. They P(B) is

(a) $\frac{2}{7}$ (b) $\frac{2}{3}$ (c) $\frac{3}{8}$ (d) $\frac{1}{8}$

11. If $P(A) = \frac{1}{2}$, $P(B) = \frac{3}{8}$, $P(A \cap B) = \frac{1}{5}$, than $P(A/B)$ is

(a) $\frac{2}{5}$ (b) $\frac{8}{15}$ (c) $\frac{2}{3}$ (d) $\frac{5}{8}$

12. In a probability distribution of a random variable 'X' the sum of all Probabilities is equal to

(a) 0 (b) -1 (c) 1 (d) Any non-negative integer.

(2 Mark Questions)

1. A die is rolled. If the outcome is an event number. What is the Probability that it is a prime?

2. If A and B are two events such that $P(A) = \frac{1}{4}$, $P(B) = \frac{1}{2}$ and $P(A \cap B) = \frac{1}{8}$. Find P(not A and not B)

3. Given that event A and A such that $P(A) = \frac{1}{2}$, $P(A \cup B) = \frac{3}{5}$ and $P(B) = p$ find p if

(i) they are mutually exclusive.

(ii) they are independent events.

4. A problem of mathematics is given to 3 students whose chances of solving it are $\frac{1}{2}, \frac{1}{3}$ and $\frac{1}{4}$. What is the probability that the problem is solved.

5. A die is tossed thrice. Find the probability of getting an odd number at least once.

6. Obtain binomial probability distribution, if $n = 6,\ P = \frac{1}{5}$.

7. If A and B are two independent events, then the probability of occurance of atleast one of A or B is given by $1 - P(\overline{A})P(\overline{B})$

8. A pair of coins is tossed once. Find the probability of showing at least one head.

9. A coin is tossed 6 times. Find the probability of obtaining no head.

10. A bag contains 5 white and 3 Black balls. Two balls are drawn at random without replacement. Determine the probability of getting both the balls black.

11. Two dice are thrown once. Find the probability of getting an event number on the first die or a total 8.

(4 Mark Questions)

1. In a class of 25 students with roll numbers 1 to 25. A student is picked up at random to answer a question. Find the probability that the roll number of the selected student is either a multiple of 5 or 7.

2. A car hit a target 4 times in 5 shots B three times in 4 shots, C twice in 3 shots. They fire a volley. What is the probability that two shots at least hit.

3. A and B throw a die alternatively till one of them throws a '6' and win the game. Find their respective probabilities of winning if A starts first.

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4. A drunkard man takes a step forward with probability 0.4 and backward with probability 0.6. Find the probability that at the end of eleven steps he is one step away from the starting point.

5. Two cards are drawn from a pack of well shuffled 52 cards. Getting an ace or a spade is considered a success. Find the probability distribution for the number of success.

6. In a game, a man wins a rupee for a six and looses a rupee for any other number when a fair die is thrown. The man decided to throw a die thrice but to quit as and when he gets a six. Find the expected value of the amount he win/looses.

7. Suppose that 10% of men and 5% of women have grey hair. A grey haired person is selected at random. What is the probability of this person being male? Assume that there are 60% males and 40% females?

8. A card from a pack of 52 cards is lost. From the remaining cards of the pack, two cards are drawn. What is the probability that they both are diamonds?

9. Ten eggs are drawn successively with replacement from a lot containing 10% defective eggs. Find the probability that there is at least are defective egg.

10. Find the variance of the number obtained on a throw of an unbiased die.

11. In a hurdle race, a player has to cross 8 hurdles. The probability that he will clear each hurdle is $\frac{4}{5}$ whats the probability that he will knock down fever than 2 hurdles.

12 Bag A contain 4 red and 2 black balls. Bag B contain 3 red and 3 black balls. One ball is transferred from bag A to bag B and then a ball is drawn from bag B. The ball so drawn is found to be red find teh probability that the transferred ball is black.

13 If a fair coin is tossed 10 times find the probability of getting.

(i) exactly six heads,

(ii) at least six heads,

.14 A card from a pack of 52 cards is lost. From the remaining cards of the pack, two cards are drawn and are found to be hearts. Find the probability of missing card to be heart.

15 A box X contain 2 white and 3 red balls and a bag Y contain 4 white and 5 red balls. One ball is drawn at random from one of the bag and is found to be red. Find the probability that it was drawn from bag Y.

16. In answering a question on a multiple choice, a student either knows the answer or guesses. Let $\frac{3}{4}$ be the probability that he knows the answer and $\frac{1}{4}$ he the probability that he guesses. Assuming that a student who guesses at the answer will be incorrect with probability $\frac{1}{4}$. What is the probability that the student knows the answer, gives that he answered correctly.

17. Two urns A and B contain 6 black and 4 white and 4 black and 6 white balls respectively. Two balls are drawn from one of the urns. If both the balls drawn are white, find the probability that the balls are drawn from urn B.

18. Two cards are drawn from a well shuffled pack of 52 cards. Find the mean and variance for the number of face cards obtained.

19. Write the probability distribution for the number of heads obtained when there coins are tossed together. Also, find the mean and variance of the probability distribution.

SAMPLE PAPER – I

CLASS – XII

MATHEMATICS

Time : 3 hrs. Theory : 90 marks

CCE : 10 marks

Total : 100 marks

1. All questions are compulsory.
2. Q.1 will consist of 10 parts and each part will carry one (1) mark.
3. Q.2 to Q.9 each will be 2 marks.
4. Q.10 to Q.19 each will be of 4 marks.
5. Q.20 to Q. 23 each will be of 6 marks.
6. There will be no overall choice. There will be an internal choice in any 3 questions of 4 marks each and all questions of 6 marks [Total of 7 internal choices]
7. Use of calculator is not allowed.

Q.1.(i) R is a relation on N given by : $N = \{(x, y)\}: 4x + 3y = 20,$ which of the following belongs to R. 1

(a) (-4, 12) (b) (5, 0)

(c) (3, 4) (d) (2, 4)

(ii) The value of tan $\left[\cos^{-1}\left(\frac{4}{5}\right) + \tan^{-1}\left[\frac{2}{3}\right]\right]$ is 1

(a) $\frac{6}{17}$ (b) $\frac{16}{7}$ (c) $\frac{7}{16}$ (d) None of these

(iii) If a+b+c=0 one root of 1

$$\begin{vmatrix} a-x & c & b \\ c & b-x & a \\ b & a & c-x \end{vmatrix} = 0$$

(a) x=1 (b) x=2 (c) $x = a^2 + b^2 + c^2$

(d) $x = 0$

(iv) Find k, $\begin{bmatrix} 2x+1 & x<2 \\ k & x=2 \\ 3x-1 & x>2 \end{bmatrix}$ to be continuous at $x = 2$ is 1

(a) 3 (b) -5 (c) 0 (d) 5

(v) The derivative $\sqrt{\log(\sin x)}$ of is : 1

(a) $\dfrac{\cot x}{2\sqrt{\log(\sin x)}}$ (b) $\cot x$

(c) $\dfrac{1}{\sqrt{\log(\sin x)}}$ (d) $\tan x$

(vi) $\int\left(\sin^{-1}\sqrt{x}+\cos^{-1}\sqrt{x}\right)dx$ is equal to 1

(a) $\dfrac{\pi}{2}x+c$ (b) $\dfrac{\pi}{4}x+c$

(c) $x+c$ (d) $\pi x+c$

(vii) The order of differential eq : 1

$\dfrac{d^2y}{dx^2}=\sqrt{1+\left(\dfrac{dy}{dx}\right)^3}$ is

(a) 2 (b) 1 (c) 3 (d) none of these

(viii) The unit vector perpendicular to the vector $\hat{i}+\hat{j}$ and $\hat{j}+\hat{k}$ are 1

(a) $\hat{i}+\hat{j}+\hat{k}$ (b) $\dfrac{1}{\sqrt{3}}\left(\hat{i}+\hat{j}+\hat{k}\right)$

(c) $\hat{i}-\hat{j}+\hat{k}$ (d) $\dfrac{1}{\sqrt{3}}\left(\hat{i}-\hat{j}+\hat{k}\right)$

(ix) The sine of the angle between the straight line 1

$\dfrac{x-2}{3}=\dfrac{y-3}{4}=\dfrac{z-4}{5}$ and the plane $2x-2y+z=5$ is

(a) $\dfrac{10}{6\sqrt{5}}$ (b) $\dfrac{4}{5\sqrt{2}}$ (c) $\dfrac{2\sqrt{3}}{5}$ (d) $\dfrac{\sqrt{2}}{10}$

(x) If P(A)= $\dfrac{1}{2}$, P(B) = $\dfrac{3}{8}$ and $P(A\cap B)=\dfrac{1}{5}$ then P(A/B) is equal to 1

(a) $\dfrac{2}{5}$ (b) $\dfrac{8}{15}$ (c) $\dfrac{2}{3}$ (d) $\dfrac{5}{8}$

Q.2. If $A=\begin{bmatrix}\sin\alpha & \cos\alpha\\ -\cos\alpha & \sin\alpha\end{bmatrix}$ then prove that A'A=I 2

Q.3. For what value of k is following functions continuous at $x=0$ 2

$$F(x)=\begin{cases}\dfrac{1-\cos 4x}{8x^2}, & x\neq 0\\ k & x=0\end{cases}$$

Q.4. Integrate $\int e^x\left(\tan^{-1}x+\dfrac{1}{1+x^2}\right)dx$ 2

Q.5. Evaluate $\int_{-\pi/2}^{\pi/2}\sin^5 x\,dx$ 2

Q.6. Solve $\dfrac{dy}{dx}=\dfrac{1+y^2}{1+x^2}$ 2

Q.7. Solve the differential eq. 2

$(1+e^{2x})dy+(1+y^2)e^x\,dx=0$ given that x=0, y=1

Q.8. Find the value of λ so that the planes 2

$2x+\lambda y+3z=15$ and $x-y+7\lambda z=13$ are perpendicular

Q.9. A die is rolled. If the outcomes is an even number. What is the probability that it is prime. 2

Q.10. Let A=R-{3} and B=R-{1} consider the function $f:A\rightarrow B$ defined by $f(x)=\dfrac{x-1}{x-3}$ show that f is one one and onto and hence find f^{-1} 4

Q.11. Prove that $\tan^{-1}\left(\dfrac{\sqrt{1+\cos x}+\sqrt{1-\cos x}}{\sqrt{1+\cos x}-\sqrt{1-\cos x}}\right)=\dfrac{\pi}{4}+\dfrac{x}{2}$ where $0<x<\dfrac{\pi}{2}$ 4

Q.12. Let $A=\begin{bmatrix}2 & -1\\ 3 & 4\end{bmatrix}$, $B=\begin{bmatrix}5 & 2\\ 7 & 4\end{bmatrix}$, $C=\begin{bmatrix}2 & 5\\ 3 & 8\end{bmatrix}$ find a matrix D such that CD-AB=0

OR 4

Show that by using properties of determinants

$$\begin{vmatrix}a^2 & bc & c^2+ac\\ a^2+ab & b^2 & ac\\ ab & b^2+bc & c^2\end{vmatrix}=4a^2b^2c^2$$

Q.13. Discuss the continuity of the function 4

$$f(x)=\begin{cases}\dfrac{1-\cos 4x}{x^2} & x\neq 0\\ 2 & x=0\end{cases} \text{ at } x=0$$

Q.14. Using differentials find approximate value of $\sqrt{25.3}$ 4

Q.15. Evaluate $\int \sec^3 x\, dx$ 4

Q.16. Find the common area bounded by the circles 4

$x^2+y^2=4$ and $(x-2)^2+y^2=4$

Q.17. Show that the differential equation 4

$(x^2+2xy-y^2)dx+(y^2+2xy-x^2)dy=0$ is homogeneous and solve it.

Q.18. If $|\vec{a}|=3,\ |\vec{b}|=5, |\vec{c}|=7$ and $\vec{a}+\vec{b}+\vec{c}=\vec{0}$ find the angle between $\vec{a}$ and $\vec{b}$

OR 4

If , $\vec{a},\vec{b},\vec{c}$ are position vectors of the vertices A,B,C of a triangle ABC then show that area of ΔABC is $\frac{1}{2}\left|\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}\right|$

Q.19. If a fair coin is tossed 10 times find the probabilly of getting 4

(i) exactly six heads (ii) At least six heads.

OR

Two urns A and B contain 6 Black, 4 white balls and 4 Black, 6 White balls respectively. Two balls are drawn from one of the urns. If both the balls drawn are white find the probability that the balls are drawn from urn B.

Q.20. For a matrix $A=\begin{bmatrix}2 & -1 & 1\\ -1 & 2 & -1\\ 1 & -1 & 2\end{bmatrix}$ verify that 6

$A^3-6A^2+9A-4I=0$ hence find A^{-1}

OR

Find the Inverse of Matrix $A\begin{vmatrix}1 & 3 & 0\\ 0 & 2 & 1\end{vmatrix}$ by using elementary column transformation.

Q.21. Show that the height of cylinder of maximum volume which can be inscribed in a sphere of the radius R is $\frac{2R}{\sqrt{3}}$ Also find the maximum volume. 6

OR

A window is in the form of a rectangle is surrounded by a semi circular opening. The total perimeter of the window is 30 metres. Find the dimensions of the rectangular part of the window to admit maximum light through the whole opening.

Q.22. Find shortest distance between the lines 6

$$\vec{r} = (1-\lambda)\hat{i} + (\lambda-2)\hat{j} + (3-2\lambda)\hat{k}$$

and $$\vec{r} = (\mu+1)\hat{i} + (2\mu-1)\hat{j} + (2\mu+1)\hat{k}$$

OR

Find the vector equation of line passing through the point (1,2,-4) and perpendicular to the two lines

$$\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7} \text{ and } \frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$$

Q.23. Maximize and Minimize $z = x + 2y$ subject to constraints 6

$$x + 2y \le 120,\ x + y \ge 60$$

$$x - 2y \ge 0 \qquad x \ge 0, y \ge 0$$

OR

A company manufacture two types of lamps say A and B. Both lamps go through a cutter and then a finisher Lamp A requires 2 hours of cutter's time and 1 hours of the finisher's time lamp B requires 1 hour of cutter's and 2 hours of finisher's time. The cutter has 100 hours and finishers has 80 hours of time available each month. Profit on one lamp A is Rs. 7.00 and on one lamp B is 13.00 Assuming that he can sell all that he produces, how many of each type of lamps should be manufactured to obtain maximum profit.

SAMPLE PAPER – II

CLASS – XII

MATHEMATICS

Time : 3 hrs. Theory : 90 marks

CCE : 10 marks

Total : 100 marks

1. All questions are compulsory.
2. Q.1 will consist of 10 parts and each part will carry one (1) mark.
3. Q.2 to Q.9 each will be 2 marks.
4. Q.10 to Q.19 each will be of 4 marks.
5. Q.20 to Q.23 each will be of 6 marks.
6. There will be no overall choice. There will be an internal choice in any 3 questions of 4 marks each and all questions of 6 marks [Total of 7 internal choices]
7. Use of calculator is not allowed.

Q.1.(i) Let X be a family of sets and R be a relation in X defined by 'A is disjoint from B'. Then R is

(a) Reflexine (b) Symmetric (c) Anti Symmetic (d) Transtive 1

(ii) Sin $\left[\frac{\pi}{3} - \sin^{-1}\left(\frac{-1}{2}\right)\right]$ is equal to 1

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{4}$ (d) 1

(iii) If A is a 3×3 non singular matrix than det $[adj(A)]$ is equal to 1

(a) $(\det A)^2$ (b) $(\det A)^3$ (c) $\det A$ (d) $(\det A)^{-1}$

(iv) The derivative of x^6 w.r.t. x^3 is 1

(a) $6x^6$ (b) $3x^2$ (c) $2x^3$ (d) x^2

(v) Derivative of $\tan^{-1}$ (cot x) w.r.t x 1

(a) -1 (b) 1 (c) tan x (d) cot x

(vi) $\int x^2 e^{x^3} dx$ equals 1

(a) $\frac{1}{3}e^{x^3}+C$ (b) $\frac{1}{3}e^{x^2}+C$ (c) $\frac{1}{2}e^{x^3}+c$ (d) $\frac{1}{2}e^{x^2}+c$

(vii) The degree of diff eq $\left(\frac{dy}{dx^2}\right)^3+\left(\frac{dy}{dx}\right)^2+\sin\left(\frac{dy}{dx}\right)+1=0$ 1

(a) 3 (b) 2 (c) 0 (d) not defined.

(viii) The projection of vector $\hat{i}-2\hat{j}+\hat{k}$ on the vector $4\hat{i}-4\hat{j}+7\hat{k}$ is 1

(a) $\frac{5\sqrt{5}}{19}$ (b) $2\frac{1}{9}$ (c) $\frac{9}{19}$ (d) $\frac{\sqrt{6}}{19}$

(ix) The line $\frac{x-1}{1}=\frac{y-1}{2}=\frac{z-3}{0}$ and $\frac{x-2}{0}=\frac{y-3}{0}=\frac{z-4}{1}$ are 1

(a) Parallel (b) Skew (c) Coincident (d) Perpendicular

(x) Two coins are tossed four times. The number of elements in sample space is

(a) 8 (b) 4 (c) 16 (d) 36 1

Q.2. If $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 2 \\ 3 & 3 & 4 \end{bmatrix}$ Find $(adj\,A)$ 2

Q.3. Discuss the continuity of the function $f(x)=\begin{cases} \frac{|x-2|}{2-x} & x\neq 2 \\ -1 & x=2 \end{cases}$ at x=2 2

Q.4. Evaluate $\int \frac{\sin 2x\,dx}{a\cos^2 x+b\sin^2 x}$ 2

Q.5. Evaluate $\int_0^1 (3x-1)dx$ 2

Q.6. Solve $\frac{dy}{dx}=\frac{2x^3y}{(x+1)}$ 2

Q.7. Solve the differential equation $\frac{dy}{dx} = 1 + x + y + xy$ 2

Q.8. The cartesian equation of a line are $3x + 1 = 6y - 2 = 1 - z$ 2

Find the direction – ratio and write its eq.: in vector form.

Q.9. If A and B are two events such that P(A)= $\frac{1}{4}$ P(B) = $\frac{1}{2}$ and $P(A \cap B) = \frac{1}{8}$. Find P(not A and not B) 2

Q.10. Check the function $f : R \to R$ given by $f(x) = x^3 - 6x^2 + 11x - 6$ is one one or not. 4

Q.11. Prove that $\cos^{1}\left(\frac{12}{13}\right) + \sin^{-1}\left(\frac{3}{5}\right) = \sin^{-1}\left(\frac{55}{65}\right)$ 4

Q.12. If $A = \begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix}$ find X and Y such that $A^2 - XY + YI_2 = 0$ 4

OR

Show that $\begin{vmatrix} 1 & 1 & 1 \\ x^2 & y^2 & z^2 \\ yz & zx & xy \end{vmatrix} = (y - z)(z - x)(x - y)(yz + zx + xy)$

Q.13. Discuss the continuity of following function of indicated 4

$$f(x) = \begin{cases} x \sin \frac{1}{x} & x \neq 0 \\ 0 & x = 0 \end{cases} \quad \text{at} \quad x = 0$$

Q.14. Find the intervals in which the function 4

$f(x) = x^3 - 12x^2 + 36x + 17$ is (a) increasing (b) decreasing

Q.15. Evaluate $\int \frac{1}{\sqrt{9 + 8x - x^2}} dx$ 4

Q.16. Find area of smaller region bounded by ellipse 4

$\frac{x^2}{9}+\frac{y^2}{4}=1$ and straight line $2x+3y=6$

Q.17. Solve the following diff equation 4

$$\cos^2 x\frac{dy}{dx}=\tan x-y$$

Q.18. The scalar product of vector $\hat{i}+\hat{j}+\hat{k}$ with unit vector along the sum of vector $2\hat{i}+4\hat{j}-5\hat{k}$ and $\lambda\hat{i}+2\hat{j}+3\hat{k}$ is equal to 1 Find the values of λ 4

OR

For any two vector $|\vec{a}+\vec{b}|\le|\vec{a}|+|\vec{b}|$

Q.19. Find the variance of the number obtained on a throw of an unbiased die. 4

OR

A card from a pack of 52 cards is lost. From the remaining cards of the pack two cards are drawn, what is the probability that they both are diamonds.

Q.20. Solve the following system of equ. by matrix method 6

$\frac{2}{x}-\frac{3}{y}+\frac{3}{z}=10,\ \frac{1}{x}+\frac{1}{y}+\frac{1}{z}=10,\ \frac{3}{x}-\frac{1}{y}+\frac{2}{z}=13$ Where $x\ne 0,\ y\ne 0,\ z\ne 0$

OR

By using properties of determinant prove that

$$\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix}=\left(1+a^2+b^2\right)^3$$

Q.21. Show that the right circular cone of least curved surface area and given volume has altitude equal to $\sqrt{2}$ times the radius of the base 6

OR

Show that the right angle triangle of maximum area that can be inscribed in a circle is an isosceles triangle.

Q.22. Find the co-ordinate of the foot of the perpendicular and the perpendicular distance of the point (1,3,4) from the plane $2x - y + z + 3 = 0$ Find also the image of the point in the plane. 6

OR

Find the equation of plane containing the parallel lines

$$\frac{x-4}{1} = \frac{y-3}{-4} = \frac{z-2}{5} \text{ and } \frac{x-3}{1} + \frac{y+2}{-4} = \frac{z}{5}$$

Q.23. Maximize $z = 10x + 12y$ subject to be constraints. 6

$$2x + 3y \le 30,\ 3x + y \le 17,\ x, y \ge 0$$

OR

Two tailors A and B earn Rs. 150 and Rs. 200 per day respectively. A can stitch 6 shirts and 4 paints per day. While B can stitch 10 shirts and 4 pents per day. Formulate the above L.P.P. mathematically and hence solve it to minimise the labour cost to produce at least 60 shirts and 32 pants.